VORTEX GENERATION BY TOPOGRAPHY IN LOCALLY UNSTABLE BAROCLINIC FLOWS: THE CASE OF THE LABRADOR SEA

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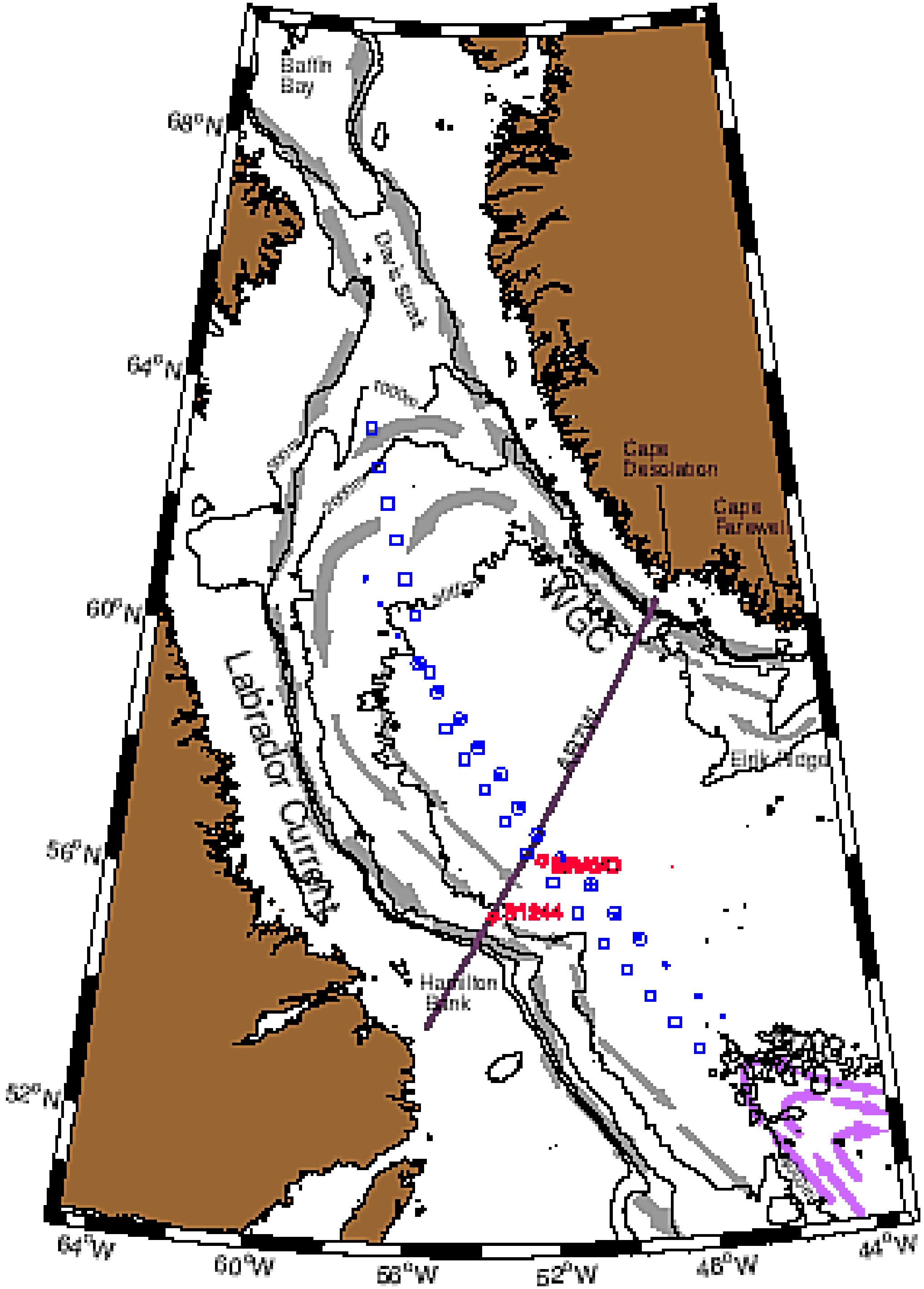
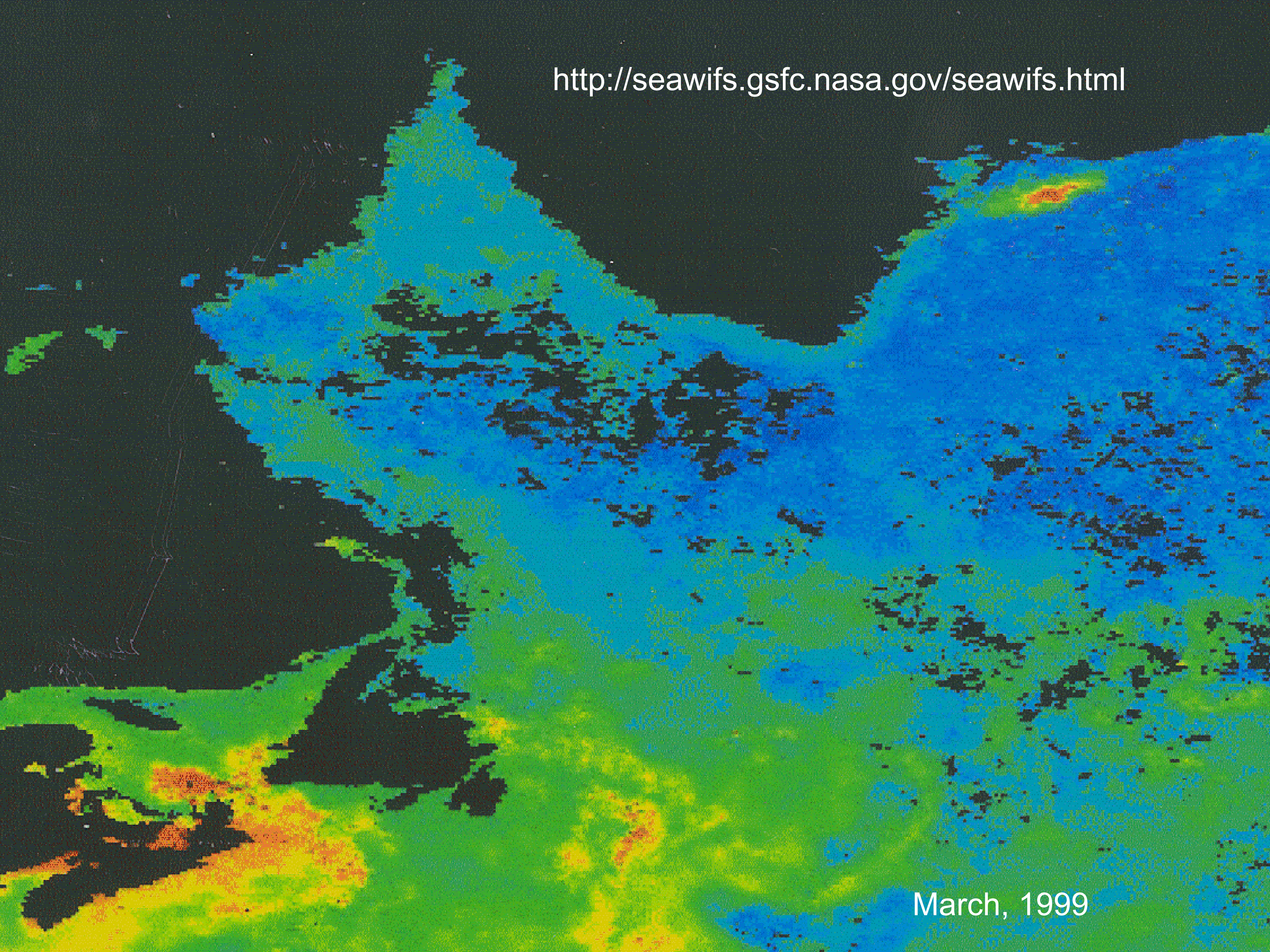
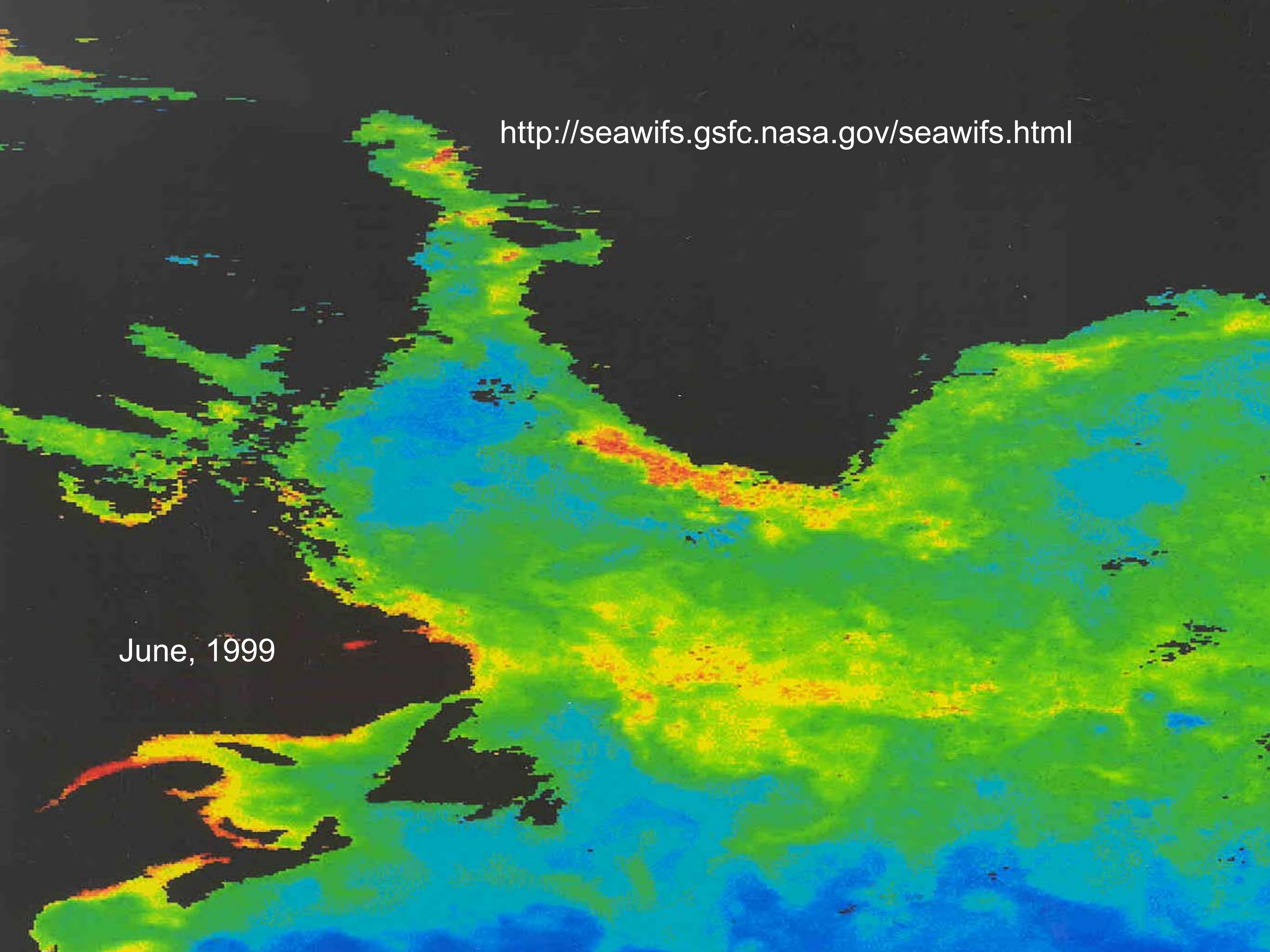
MOTIVATIONS

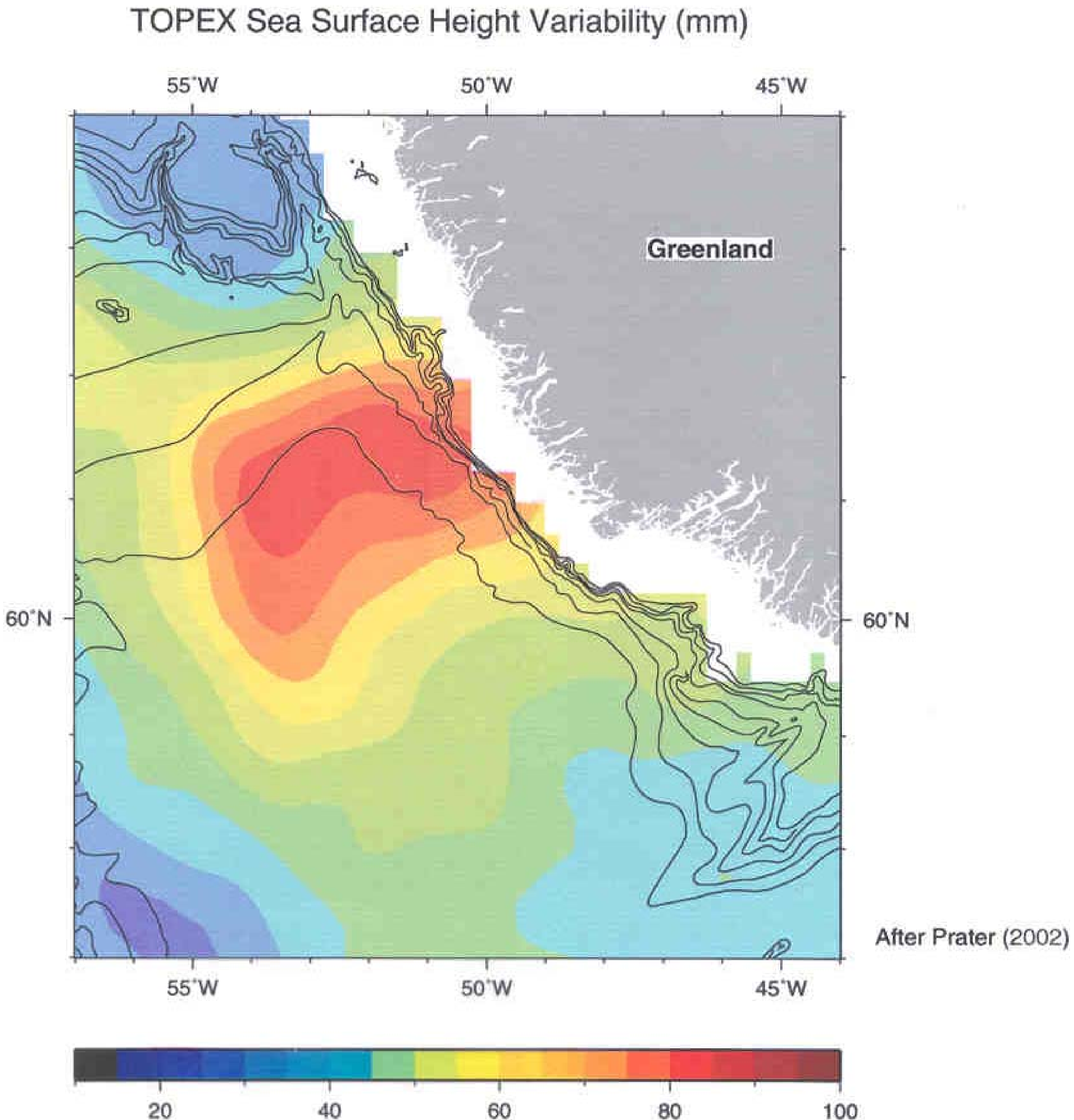
- To study the stability properties of bottom intensified boundary currents in the ocean (and of mean winds in the atmosphere) in presence of coastal mountains
- To extend the model of Samelson and Pedlosky (JFM, 1990) to a flow confined in a channel
- To 'mimic' the dynamics of the boundary currents in the eastern side of the Labrador Sea



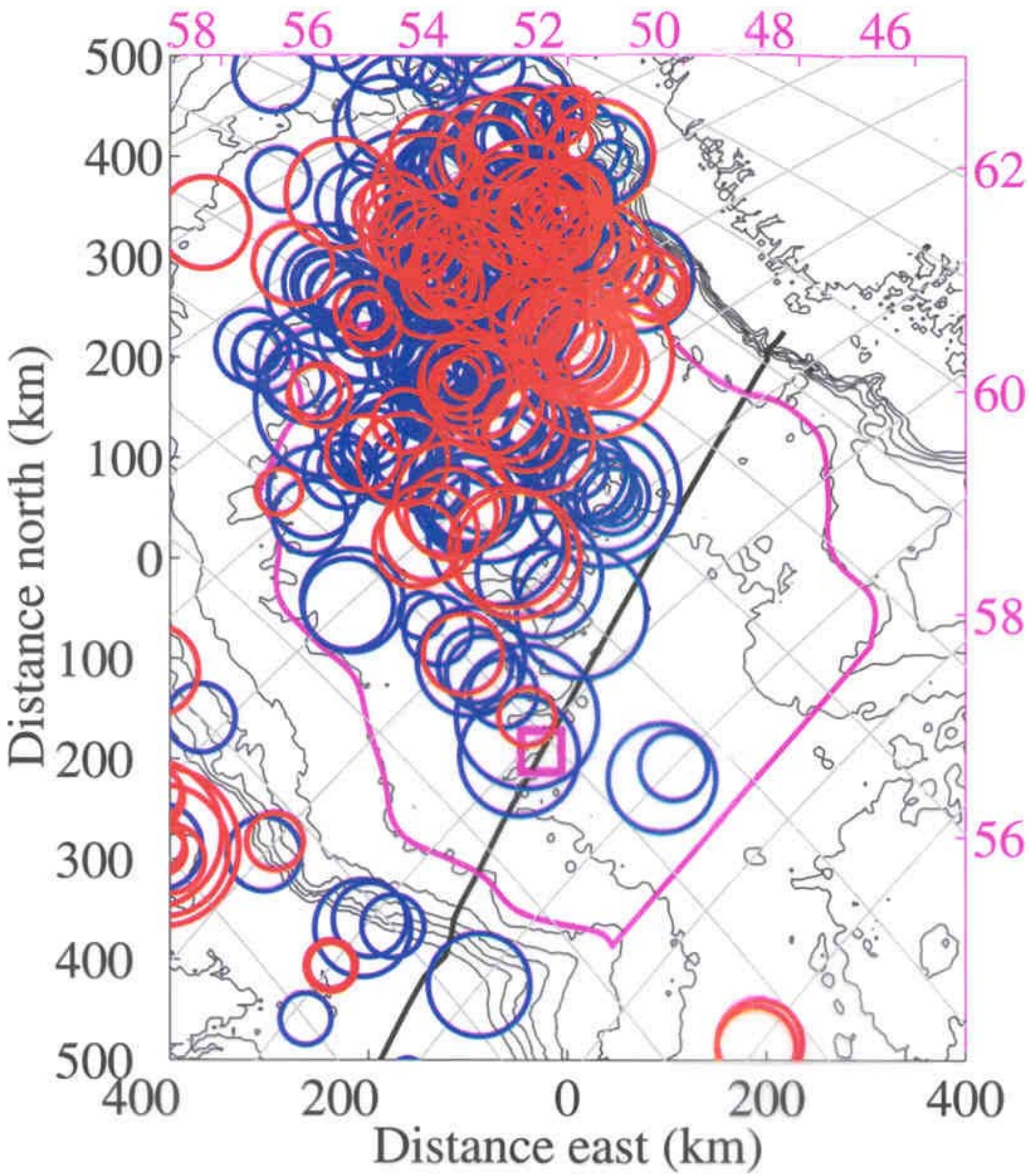
Labrador Sea current system


http://seawifs.gsfc.nasa.gov/seawifs.html
March, 1999


http://seawifs.gsfc.nasa.gov/seawifs.html
June, 1999



Confirmed by surface drifters (Fratanton, 2001) and sub-surface Palace floats (Lavender, 2001) as deep as 1500m

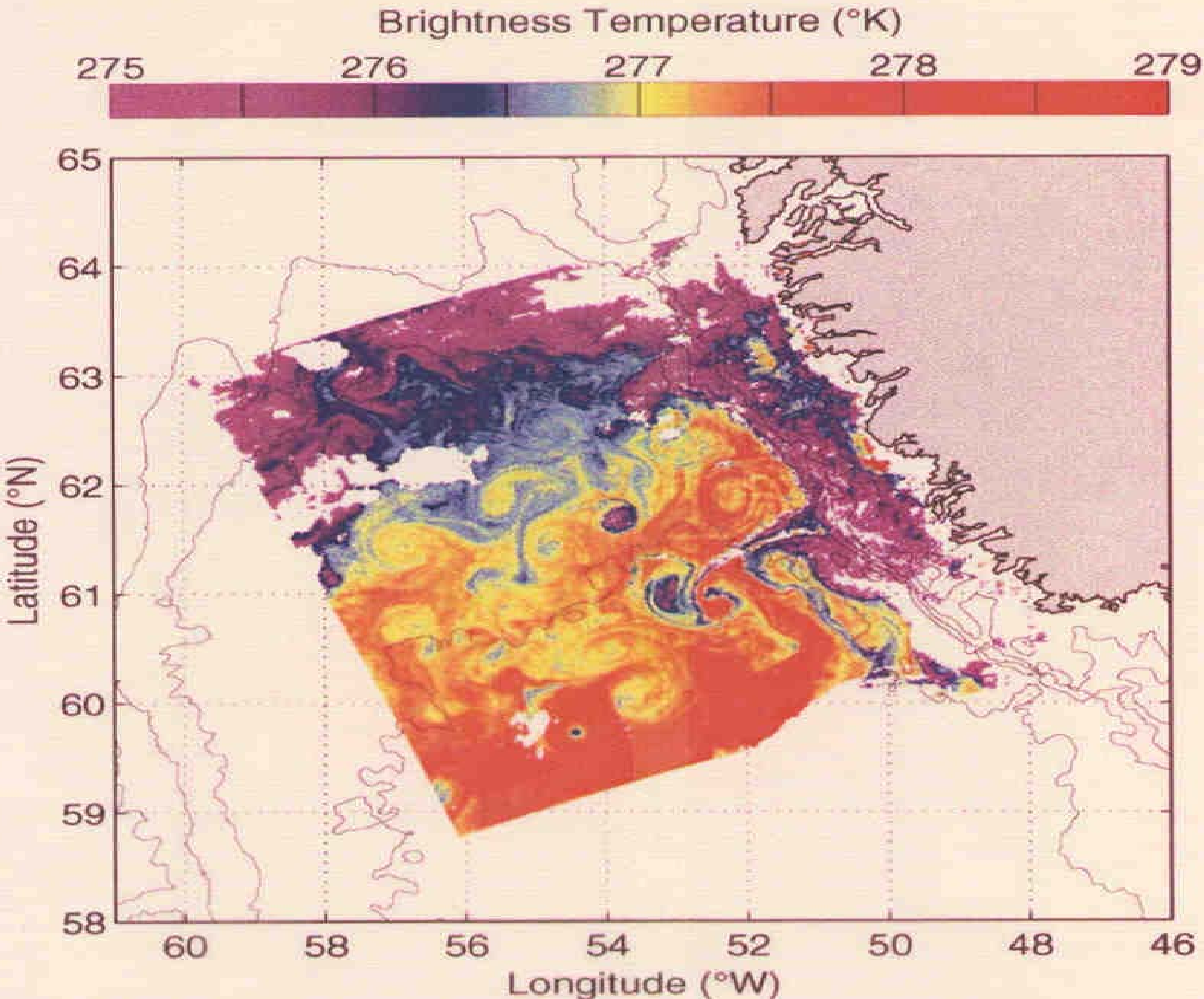


Location/size of all the eddies observed using TOPEX data between ’94 and ’99.

Lilly et al, Progress in Oceanography

change in the vortex population around 1996

- pre-1996: convective lenses, mainly AC, radius of ~ 10-12 km, formed in the middle of the basin
- post-1996: eddies form along the coast of Greenland, bigger (R~30-35km), barotropic structure with double core, move into the interior as AC or dipoles and are responsible for the restratification of the Labrador Sea after deep convection


Brightness Temperature (°K)
275
276
277
278
279
65
64
63
62
61
60
59
58
Latitude (°N)
60
58
56
54
52
50
48
46
Longitude (°W)

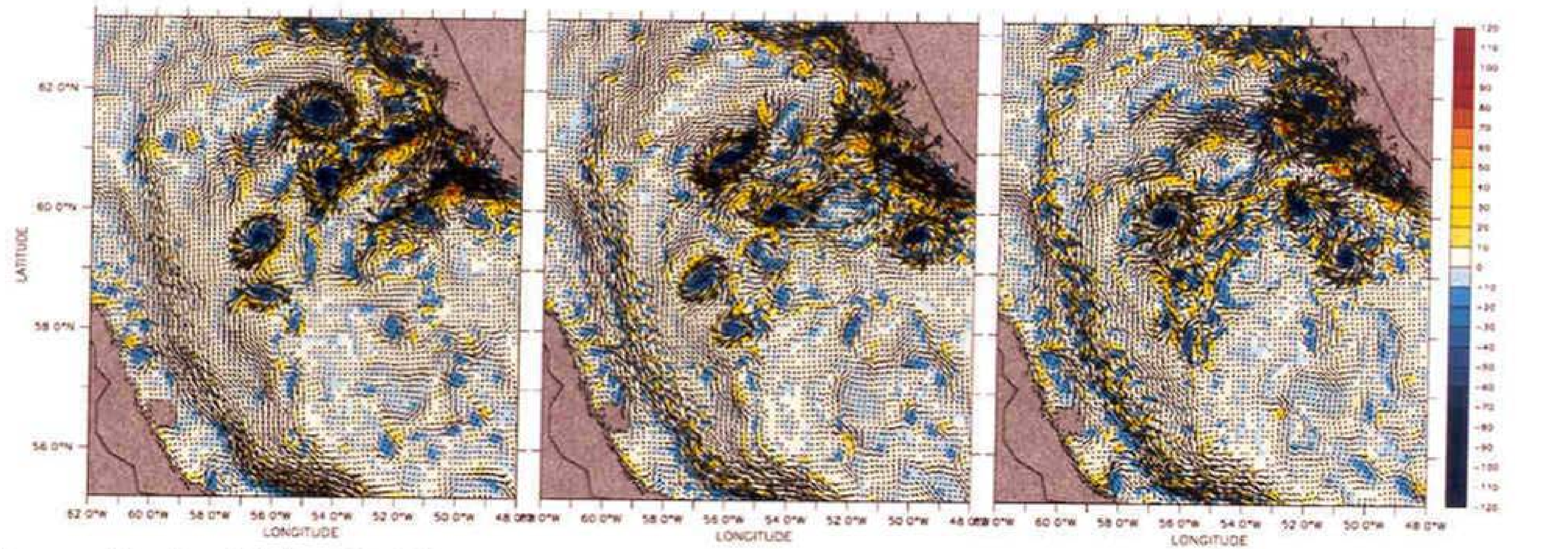
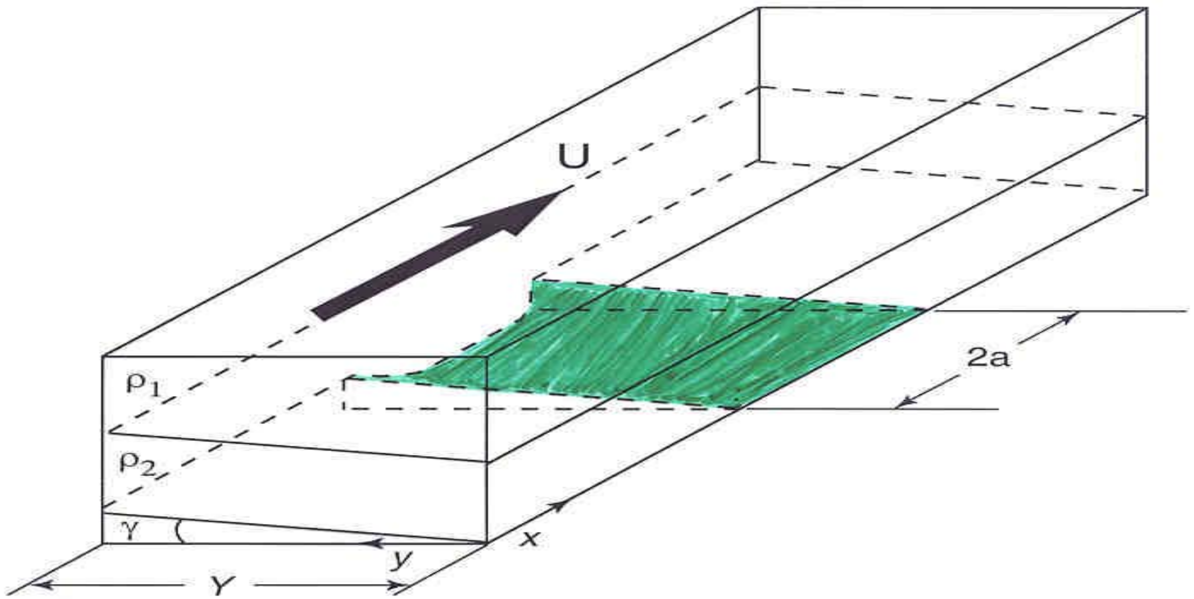


FIG. 8. A sequence of snapshots of relative vorticity (10^{6} s^{-1}) and velocity (arrows) in 107-m depth in the Labrador Sea in steps of 12 days, starting at 16 Jan. Note that only every second grid point is shown for velocities.

.. and from numerical simulations
Eden and Boning, JPO, 2003

The channel model set-up



QG flow

$\Psi_1 = \Phi_1 - Uy$
$\Psi_2 = \Phi_2$

QG approximation

A flow is nearly geostrophic if

- Horizontal accelerations are small compared to the Coriolis term → $R_o=U/(f_oL)<<1$
- Variation if f are small on the horizontal scale of the flow → $\beta L/f_o<<1$
- Fractional variations in total depth H are small → $|h'|/H<<1$, where $h=H=h'(x,y)$

2-layer QG equations

$$\frac{\partial Q_i}{\partial t} + J(\psi_i, Q_i) = \nu \nabla^4 \psi_i$$

$$Q_1 = \nabla^2 \psi_1 - F_1(\psi_1 - \psi_2) + \beta y$$

$$Q_2 = \nabla^2 \psi_2 - F_2(\psi_2 - \psi_1) + \beta y + h(x, Y)$$

Assume H1=H2=H

$L_R = (g'H)^{½}/f_o$, U and L_R used to scale horizontal length, velocities and time

↓

$F_1 = F_2 = (f_o^2 L_R^2) / (g'H) = 1$

we consider

$$\psi_1(x,y,t) = -Uy + \phi_1(x,y,t)$$

$$\psi_2(x,y,t) = \phi_2(x,y,t)$$

$$\frac{\partial q_1}{\partial t} + J(\phi_1, q_1) + \frac{\partial \phi_1}{\partial x}(\beta + U) + \frac{\partial q_1}{\partial x}U = \nu\nabla^4\phi_1$$

$$\frac{\partial q_2}{\partial t} + J(\phi_2, q_2) + \frac{\partial \phi_2}{\partial x}(\beta + \gamma - U) - \frac{\partial \phi_2}{\partial y}\frac{d\gamma}{dx}y = \nu\nabla^4\phi_2$$

where

$h(x,y)=\gamma(x)y$

and

$\gamma(x)=c-b\{\tanh[(x+a)/\sigma]-\tanh[(x-a)/\sigma]\}/2\tanh(a/\sigma)$

for $\sigma\rightarrow\infty$, $\gamma(x)$ = 

Carnevale et al., 1999

therefore…

$$Q_1 = q_1 + \beta y + Uy$$

$$Q_2 = q_2 + \beta y - Uy + \gamma(x)y$$

$$where$$

$$q_1 = \nabla^2\phi_1 - (\phi_1 - \phi_2)$$

$$q_2 = \nabla^2\phi_2 - (\phi_2 - \phi_1)$$

in 1and 3 $Q_2 = q_2 - Uy + \gamma y$

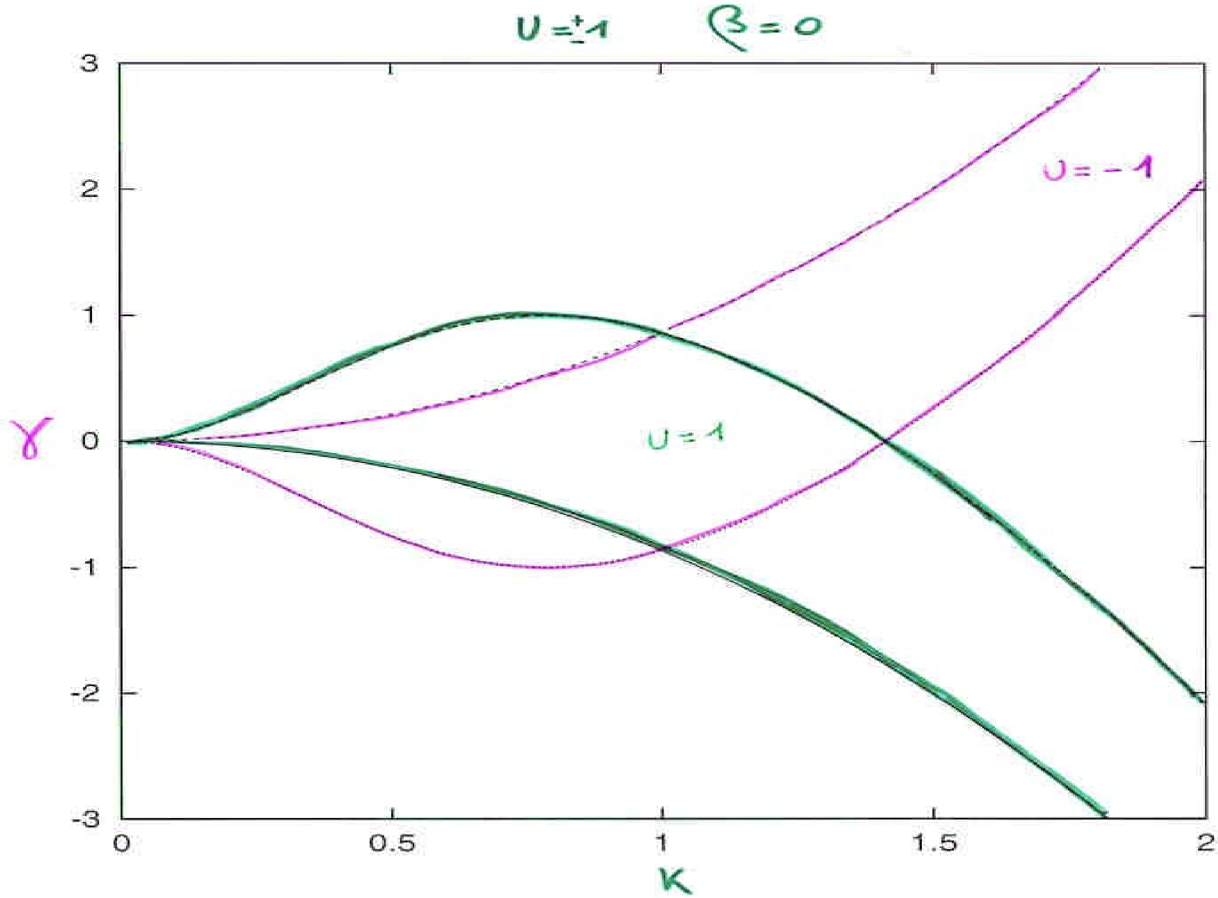
in 2 $Q_2 = q_2 - Uy$

$$\frac{\partial q_2}{\partial t} = +\gamma(x)y = -\frac{\partial \phi_2}{\partial x}(\gamma - U) \Rightarrow +\frac{\partial \phi_2}{\partial x} = v_2 < 0$$

$$\frac{\partial q_2}{\partial t} = -\gamma(x)y = -\frac{\partial \phi_2}{\partial x}(\gamma - U) \Rightarrow +\frac{\partial \phi_2}{\partial x} = v_2 > 0$$

$$\Phi_1=A_1e^{i(kx-\omega t)}\,;\;\; \Phi_2=A^2e^{i(kx-\omega t)}$$

disp. relation: $k(k^2+2)\omega^2+[-k^2(k^2+2)U+$
$+(k^2+1)(\gamma+2\beta)]\omega+k(\beta-Uk^2)(\gamma+\beta-U)$



The 1D problem

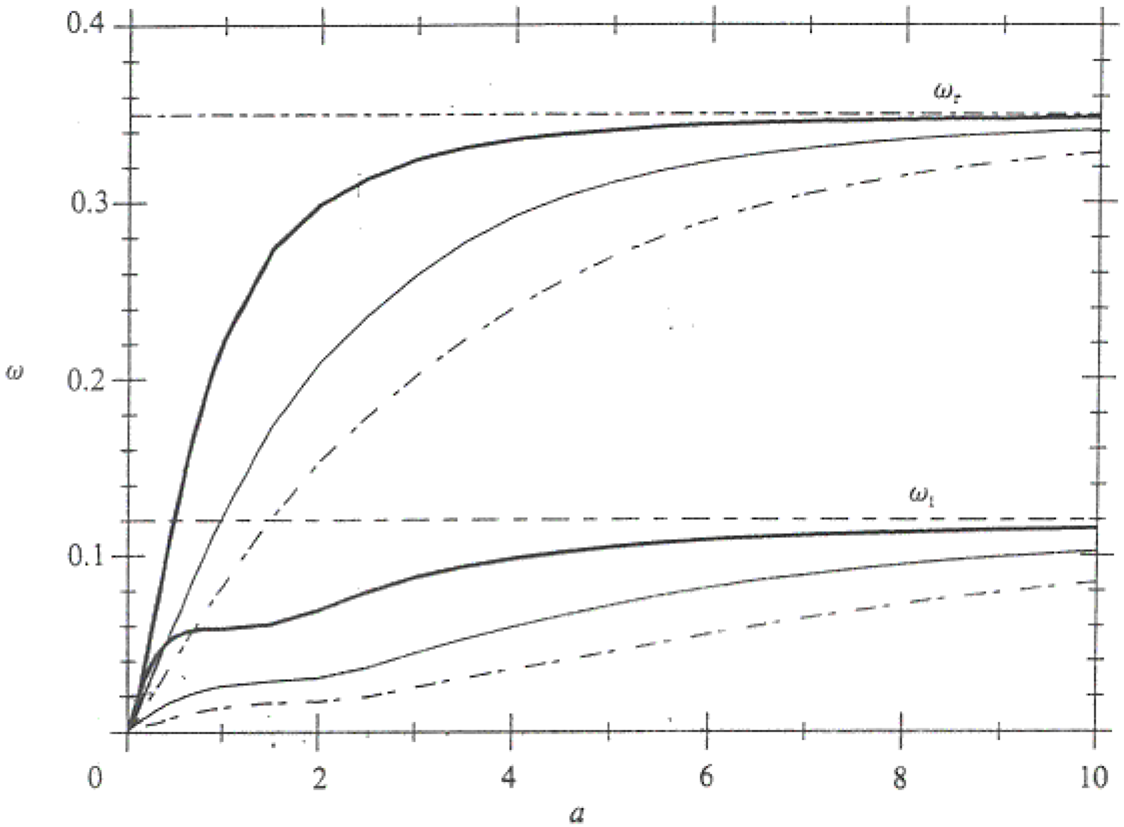
Samelson and Pedlosky, JFM, 1990

- $\Phi_1(x)=\sum_{j=1,4} A_{1j}\, e^{\,i(k_j x-\omega t)}$
- $\Phi_2(x)=\sum_{j=1,4} A_{2j}\, e^{\,i(k_j x-\omega t)}$

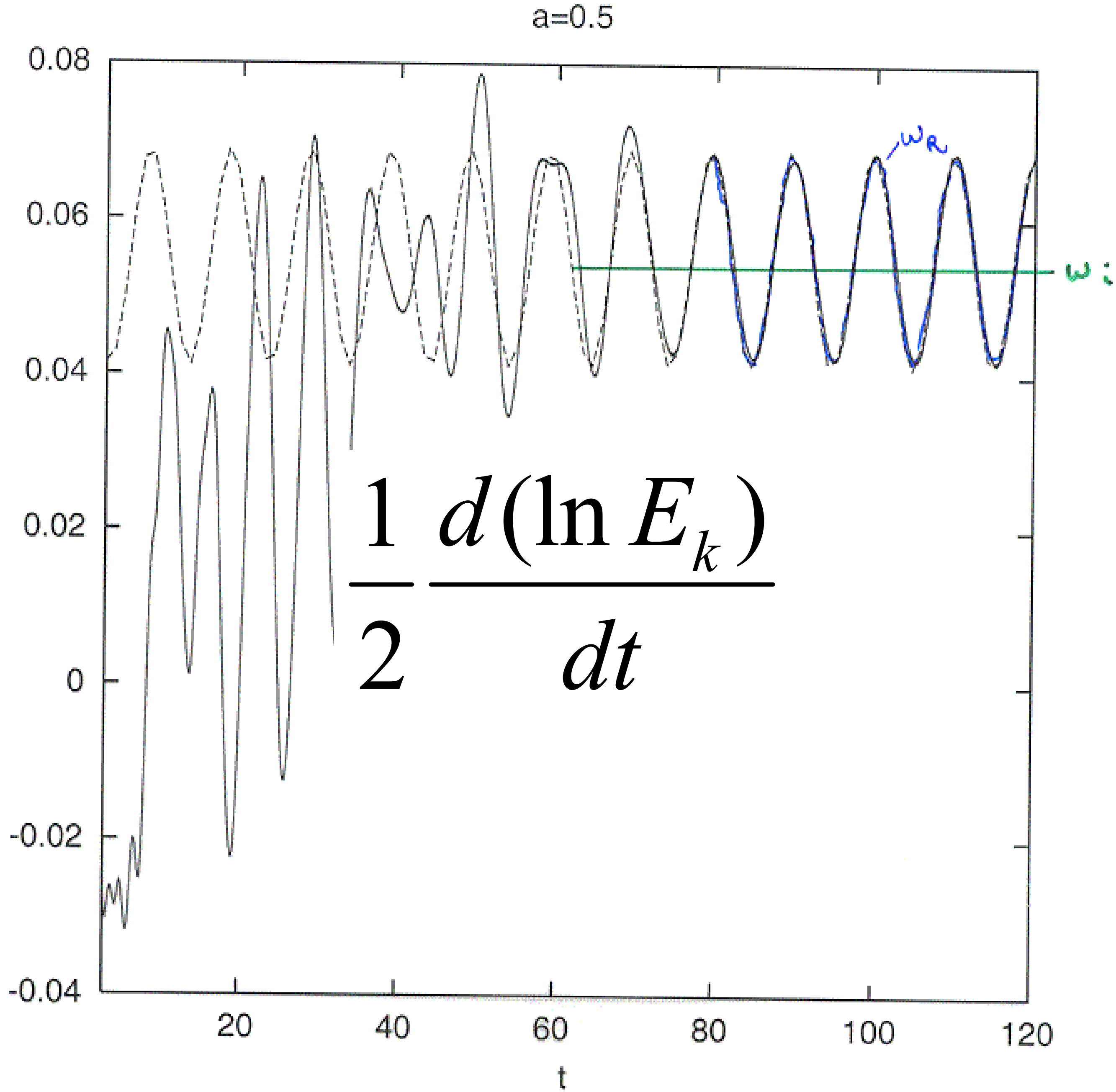
Matching conditions

1. Φ_1,Φ_2 continuous at $x = \pm a$
2. Φ_{1x}, Φ_{1xx}, Φ_{2x} continuous at $x = \pm a$
3. Φ_1, $\Phi_2 \rightarrow 0$ for $|x| \rightarrow \infty$

8 X 8 matrix eigenvalue problem for $\omega=\omega_R+i\omega_i$ and A_{1j}, A_{2j} in each region



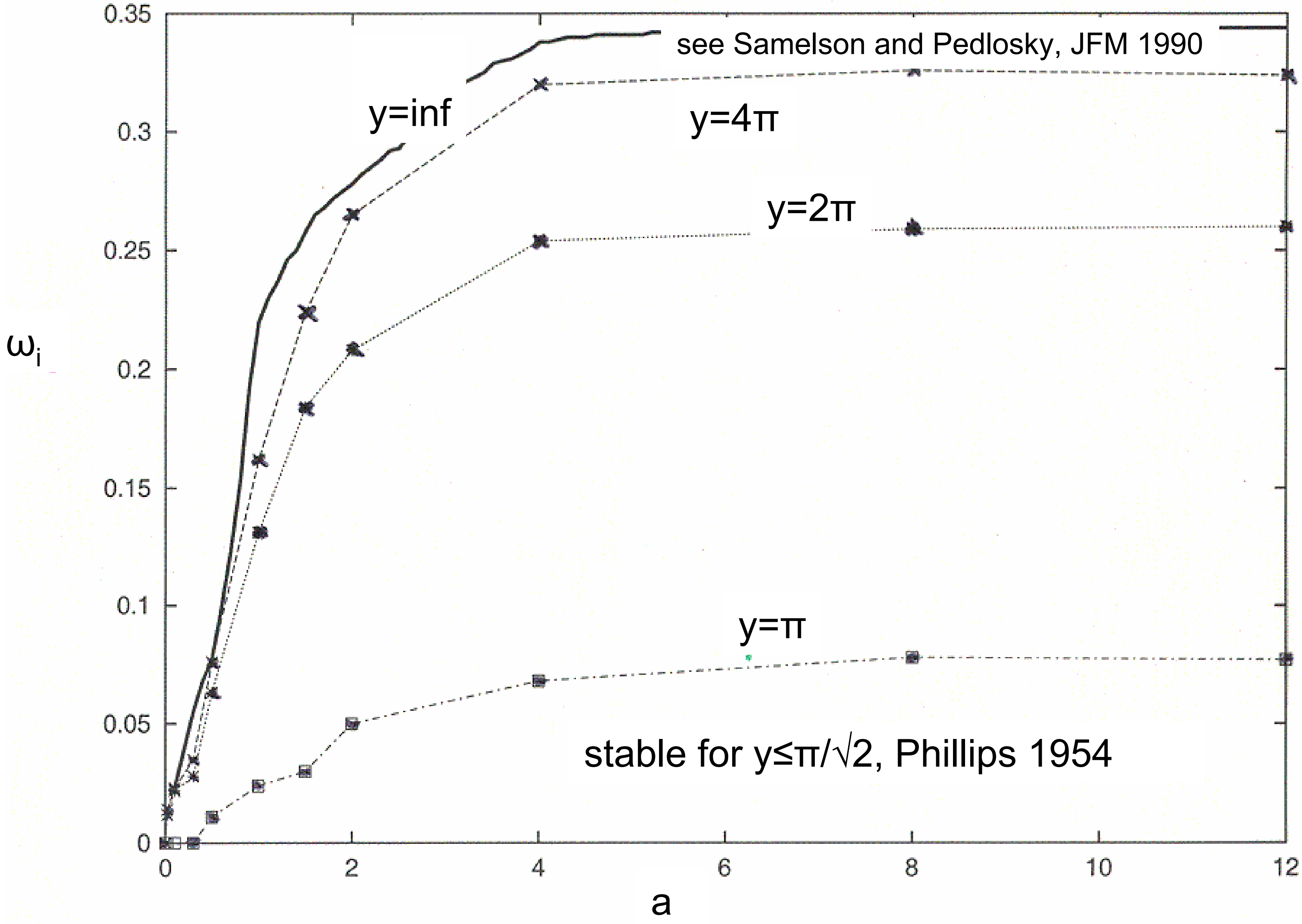
RE 3. Frequency ω_r and growth rate ω_i versus interval half-length a for local instability modes $U = 1$, $\beta = 0.25$, $\alpha_u = 0$, $\alpha_s = 2$. ——, Mode 1; ——, mode 2; —·—, mode 3; horizontal , WKB result.



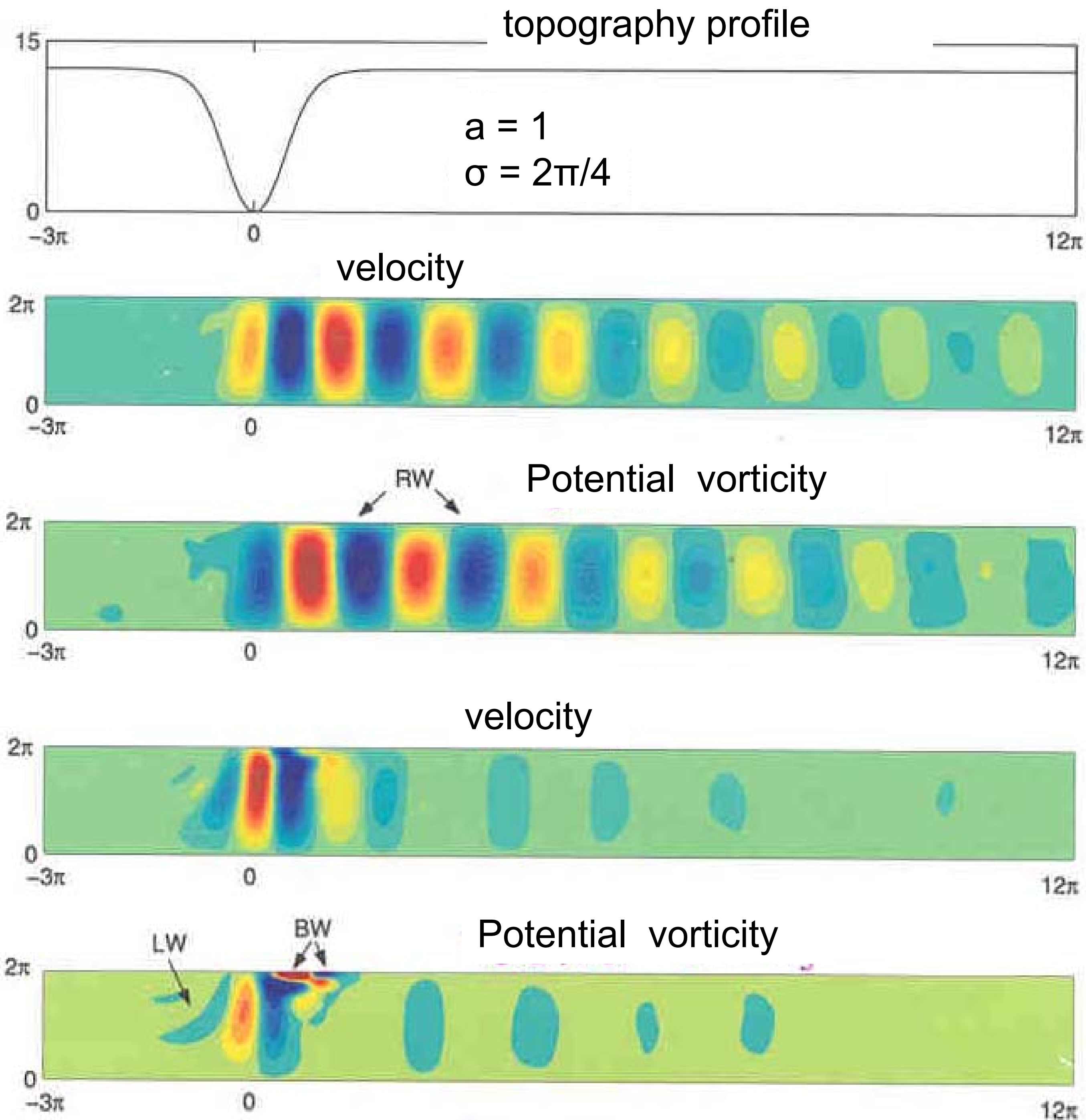
$$\frac{1}{2}\frac{d(\ln E_k)}{dt}$$

Linear modes

$E_k = C\ e^{-2i\omega t}$



see Samelson and Pedlosky, JFM 1990
y=inf
y=4π
y=2π
y=π
stable for y≤π/√2, Phillips 1954
ω_i
a
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12



layer 1

$a=1$

$\beta=0.04$

$\sigma=2\pi/4$

layer 2

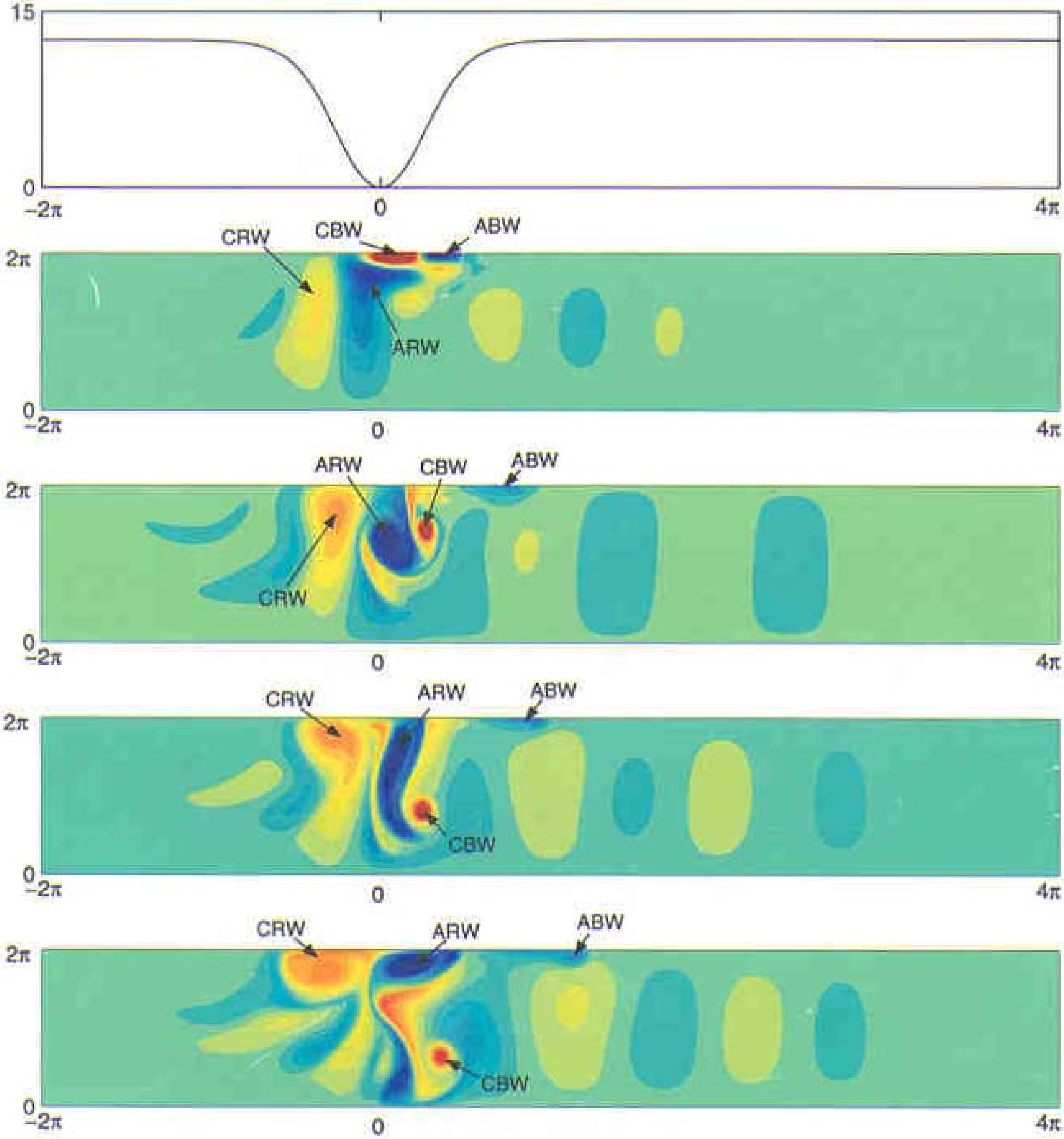
- Rossby wave $k \approx (\beta/U)^{1/2}$

- Long baroclinic wave

$k \approx - \omega (2\beta+\gamma)/[\beta(\beta+\gamma-U)]$; $A_2 \approx - A_1 \beta/(\beta+\gamma)$

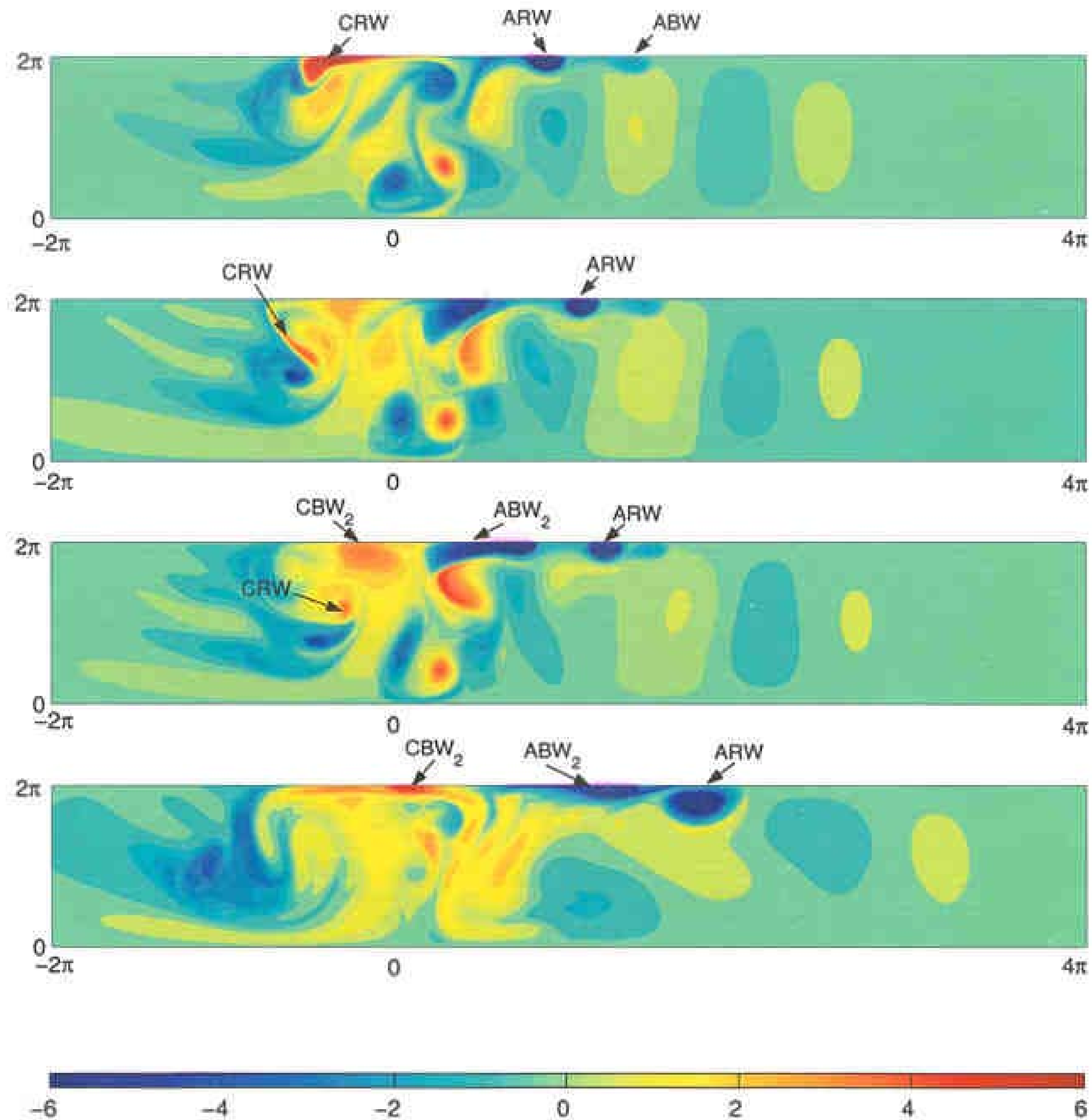
- Short bottom trapped wave

$k \approx - (\beta+\gamma-U)/\omega$; $A_2 \approx A_1 (\beta+\gamma-U)^2/\omega^2$



Potential vorticity perturbation

a=1
β=0.04
σ=2π/4

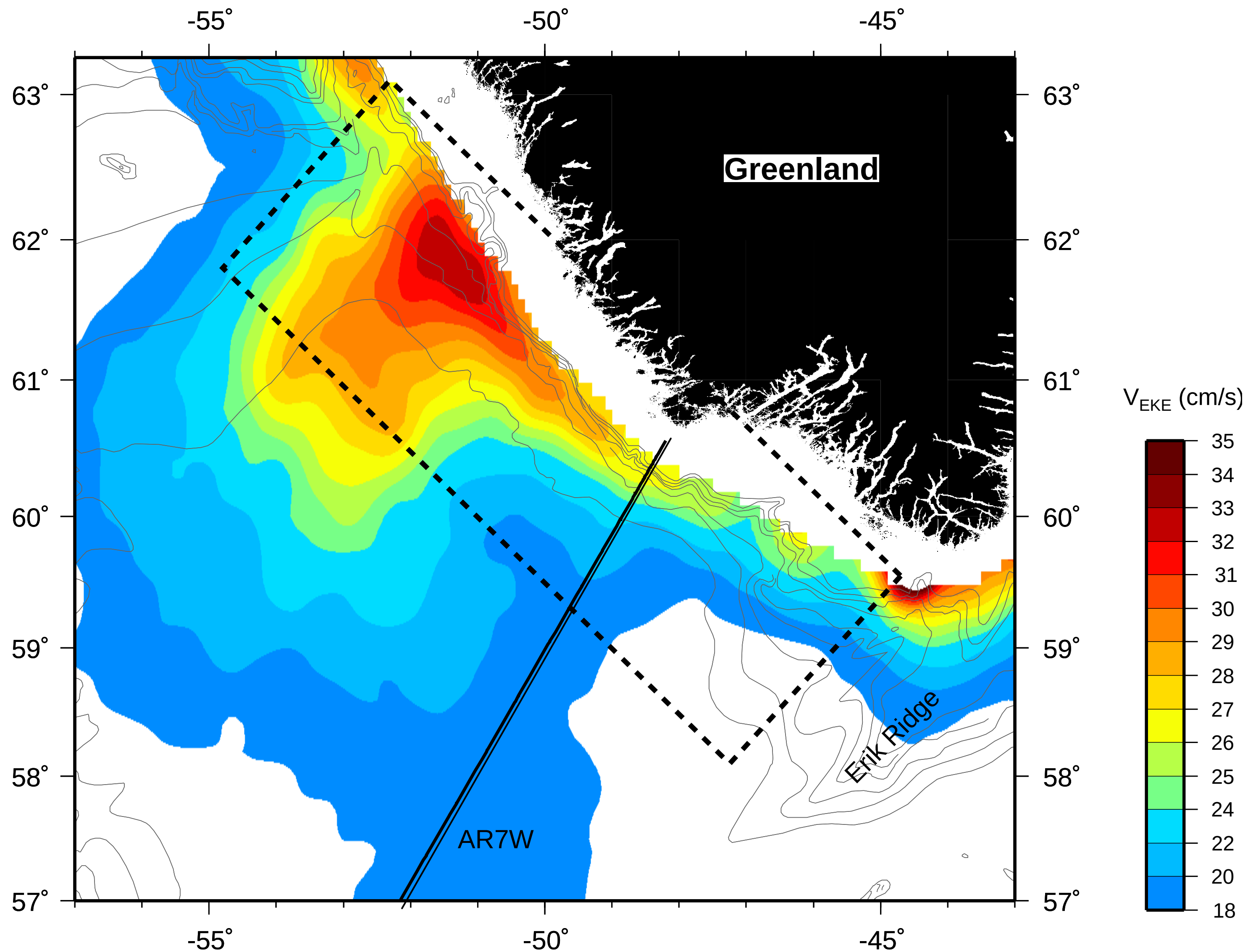

CRW
ARW
ABW
2π
0
−2π
0
4π
CRW
ARW
2π
0
−2π
0
4π
CBW$_2$
ABW$_2$
ARW
CRW
2π
0
−2π
0
4π
CBW$_2$
ABW$_2$
ARW
2π
0
−2π
0
4π
−6
−4
−2
0
2
4
6

Summary I

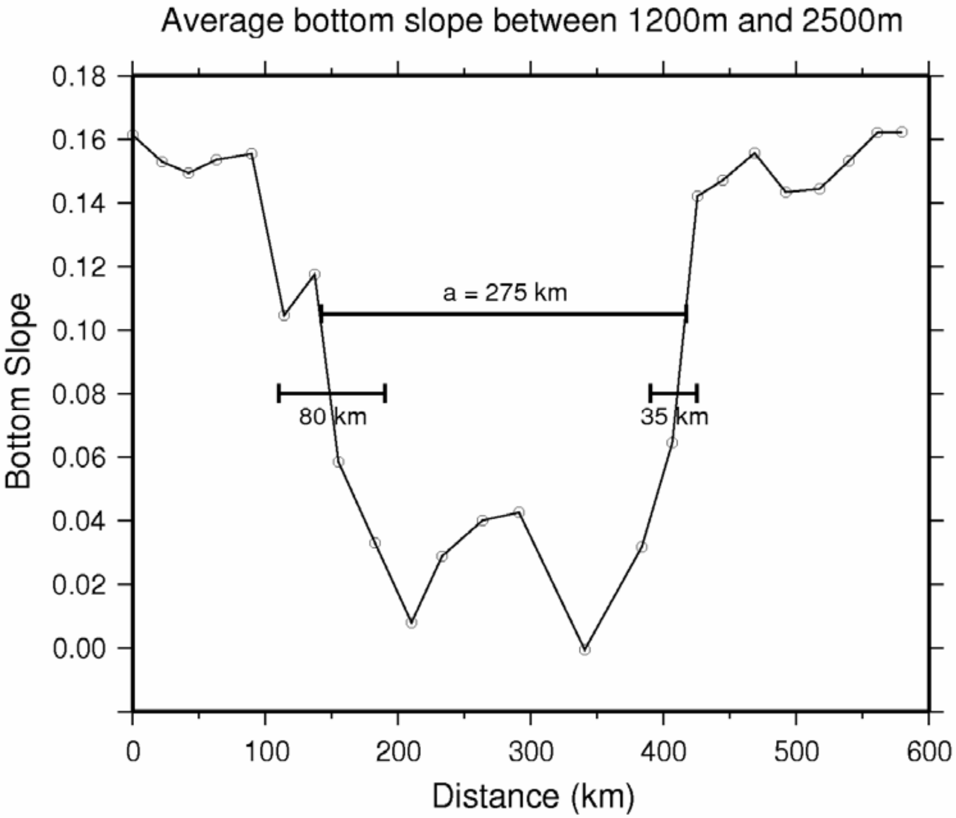
- **The bottom-trapped wave is responsible for the persistence of the instability and for the vortex formation, NO MATTER HOW SHORT IS THE INTERVAL OF INSTABILITY**
- **Only <u>local maxima in supercriticality</u> are required for the existence of unstable modes**
- **The bottom-trapped disturbance grows to balance the variation in time of relative vorticity with the ambient gradient of potential vorticity. Its confinement relies on the interaction between the zonal component of the perturbation velocity and the zonal gradient of the bathymetry (which increases with latitude → localization)**

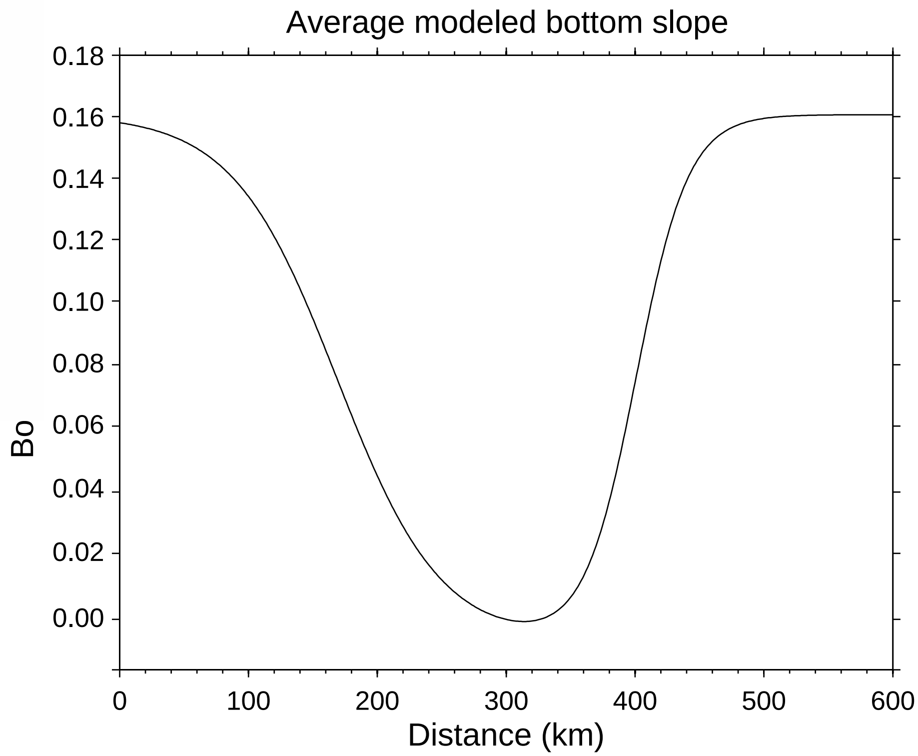
To be bit more realistic and get closer to the Labrador Sea configuration

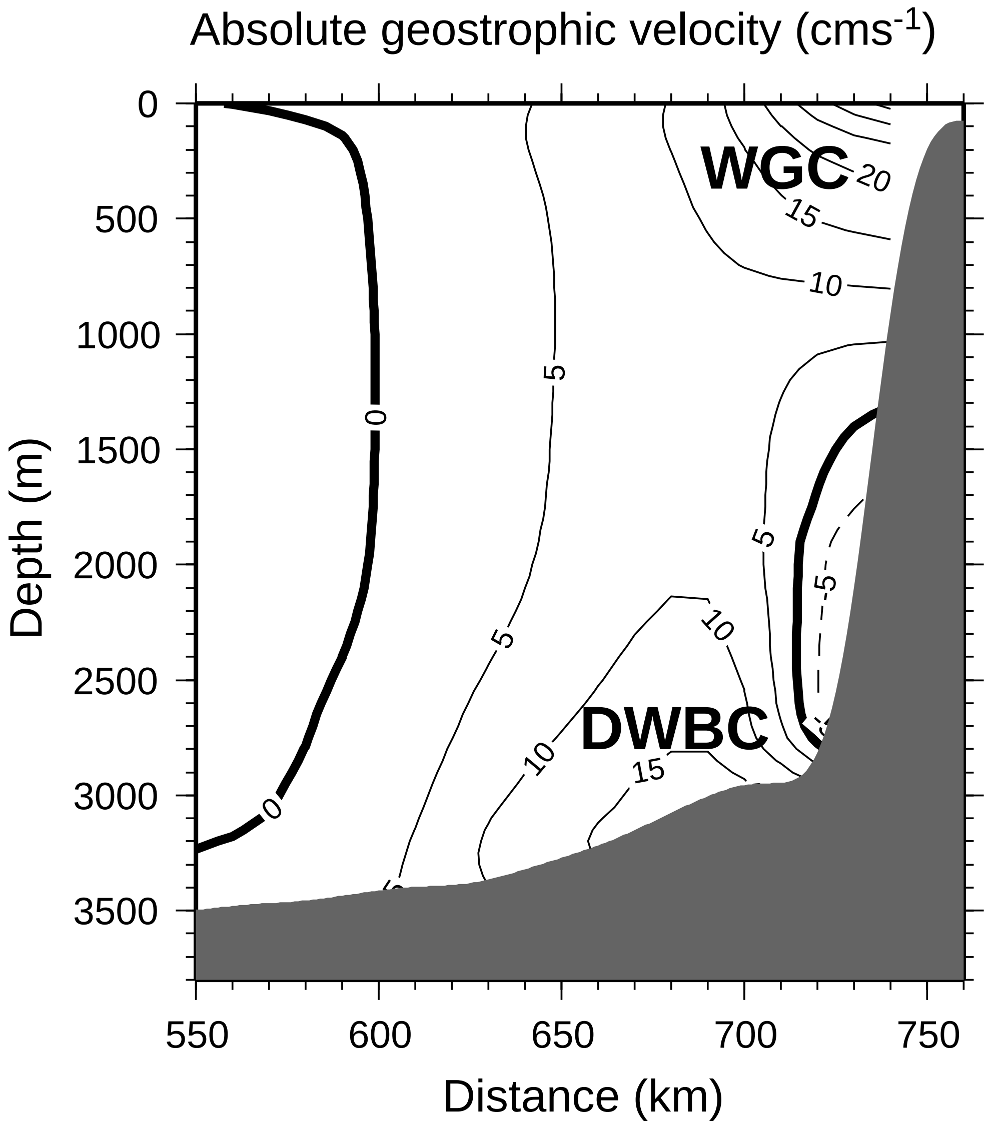
- Laterally nonuniform vertical shear → boundary confined currents
- Shear profile similar to the one observed in the Labrador Sea



surface eddy speed + WOCE AR7W hydrography line

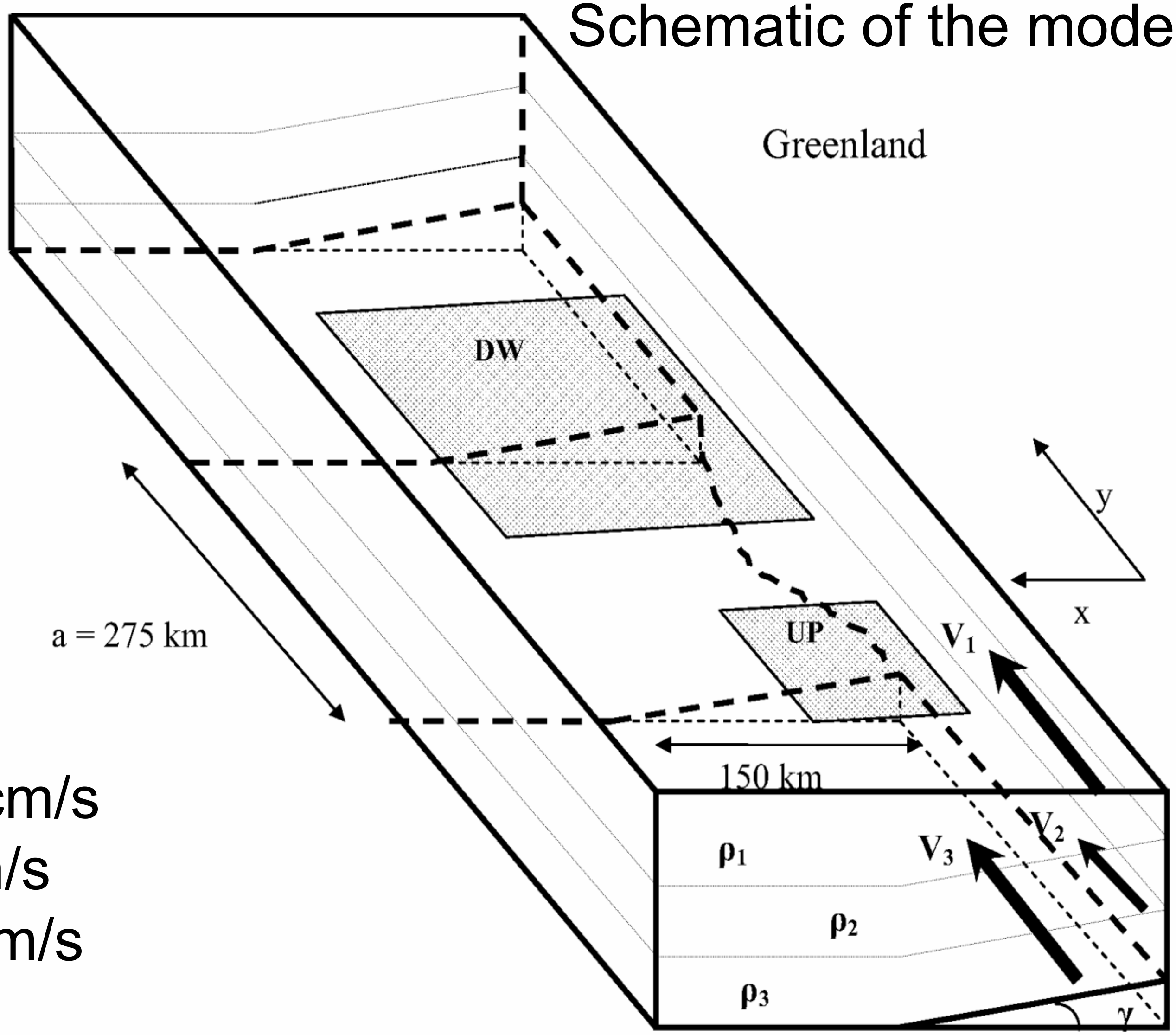

Average bottom slope between 1200m and 2500m
Bottom Slope
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
a = 275 km
80 km
35 km
0
100
200
300
400
500
600
Distance (km)


Average modeled bottom slope
Bo
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
100
200
300
400
500
600
Distance (km)



Average velocity of the BC system along the AR7W line

Schematic of the model geometry



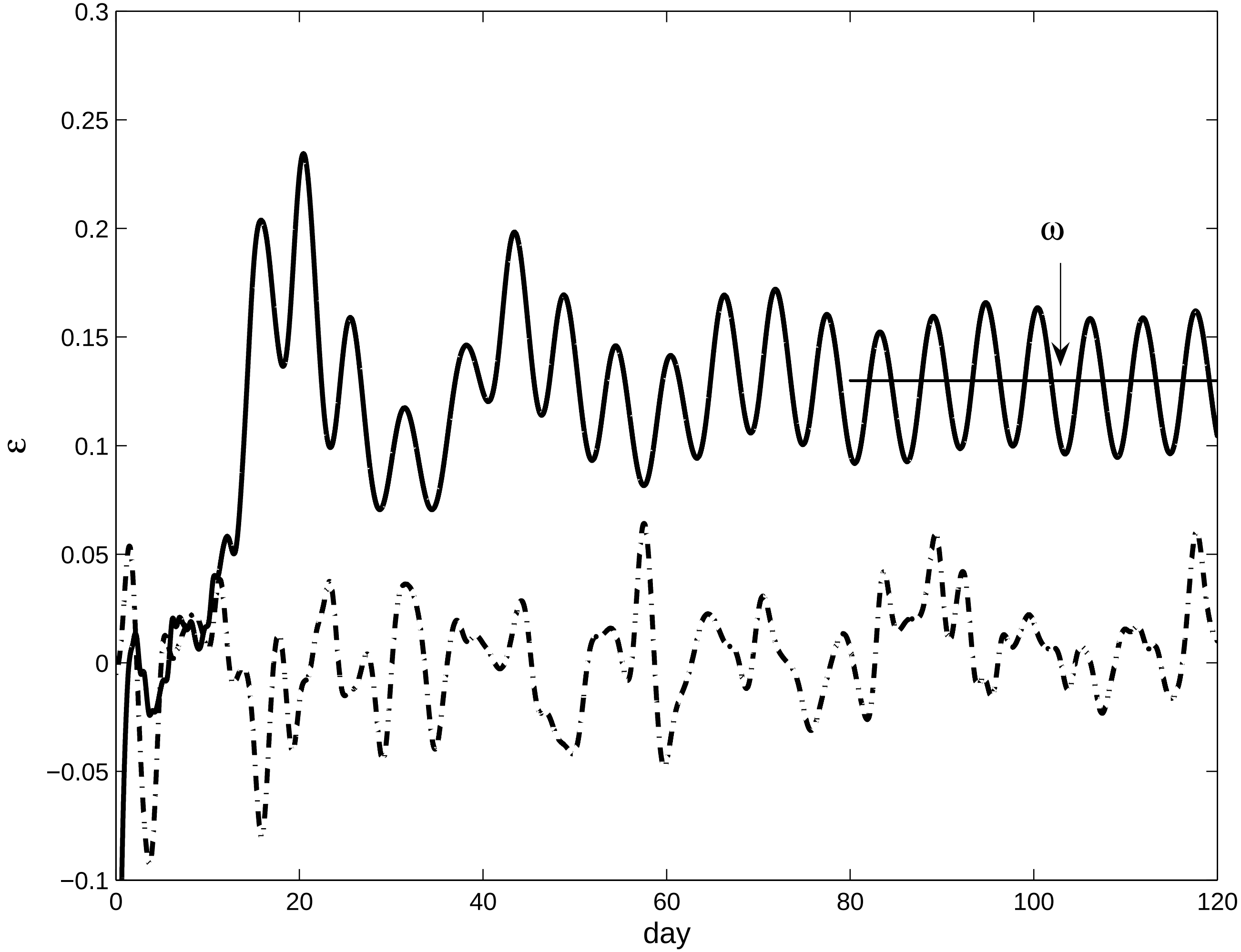
A_1=12 cm/s
A_2=6cm/s
A_3=10cm/s

$$\psi_i(x,y,t) = \frac{A_i}{\lambda} e^{-\lambda x} + \phi_i(x,y,t), \qquad i = 1,2,3, \qquad \lambda^{-1} = 60km$$

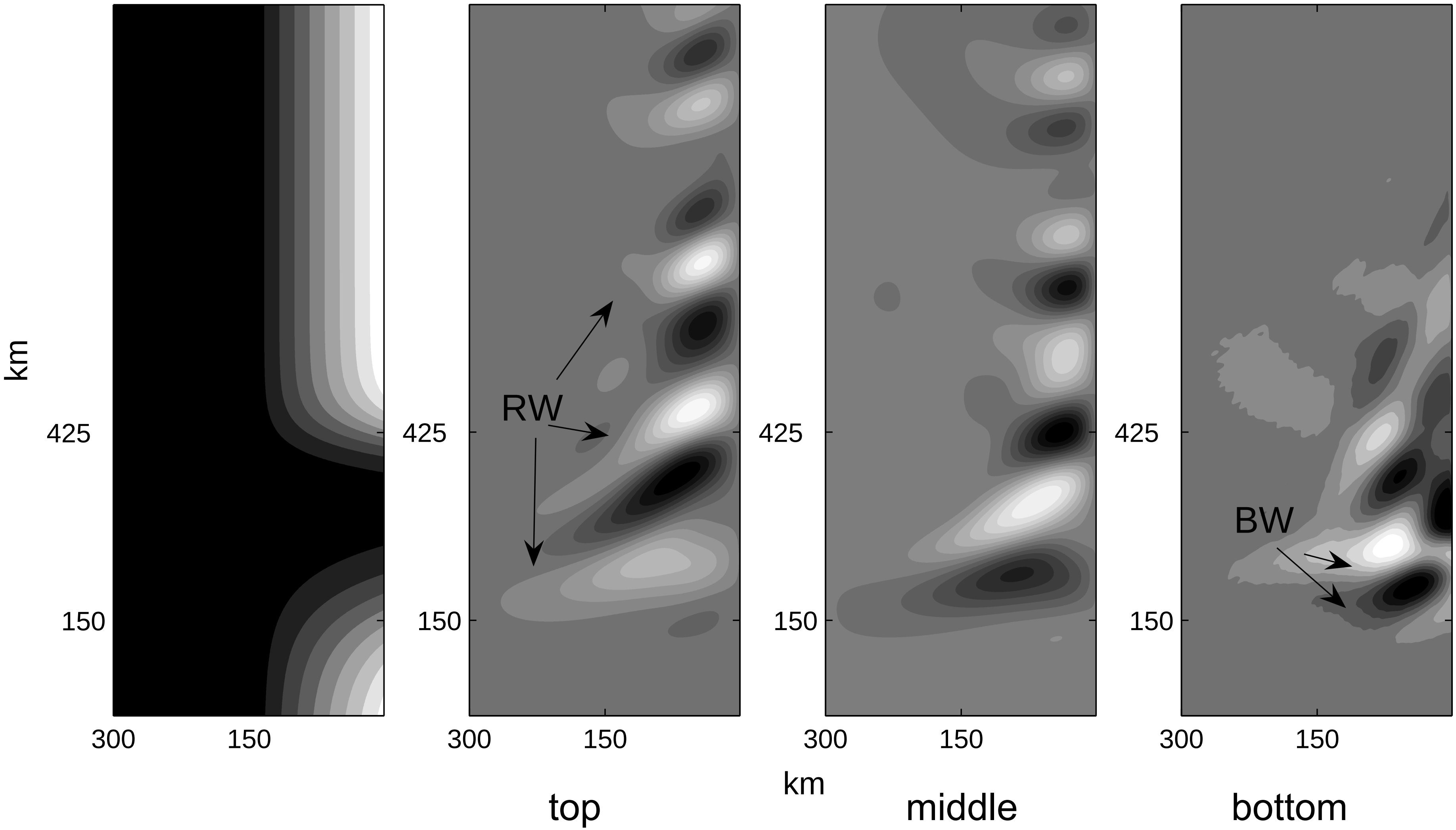
Growth rate for the linear system: 3-Layer case (solid) and barotropic model (dashed; see Carnevale et al., 1999). Condition for BAROCLINIC instability:

$$\frac{\partial q_3}{\partial x} = \left(A_3 \lambda^2 - A_3 + A_2 \right) + \gamma(y)$$

MUST change sign from + to -



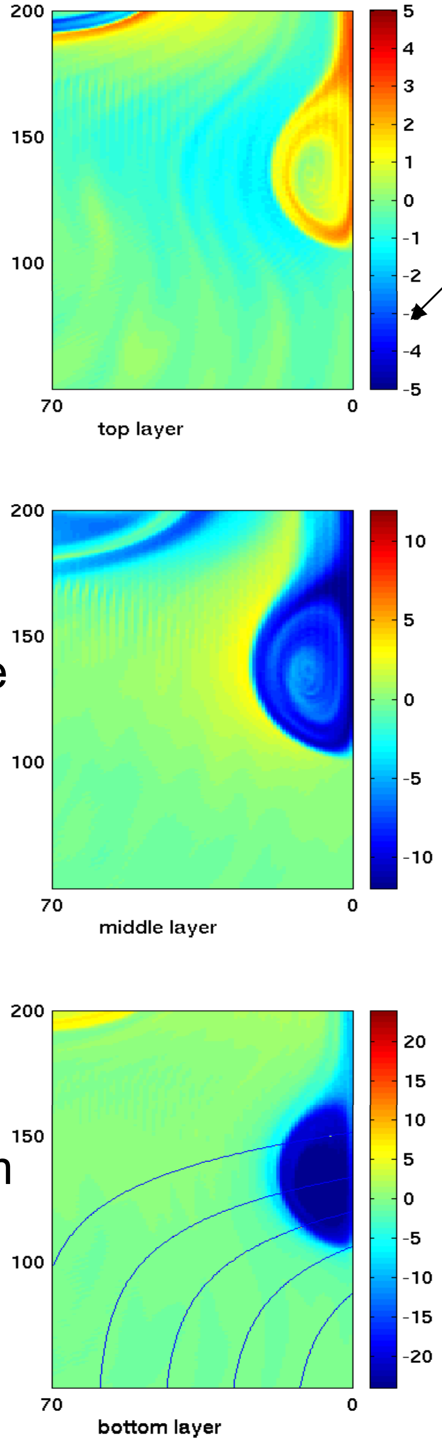
Linear solution: Potential vorticity perturbation



top

middle

bottom



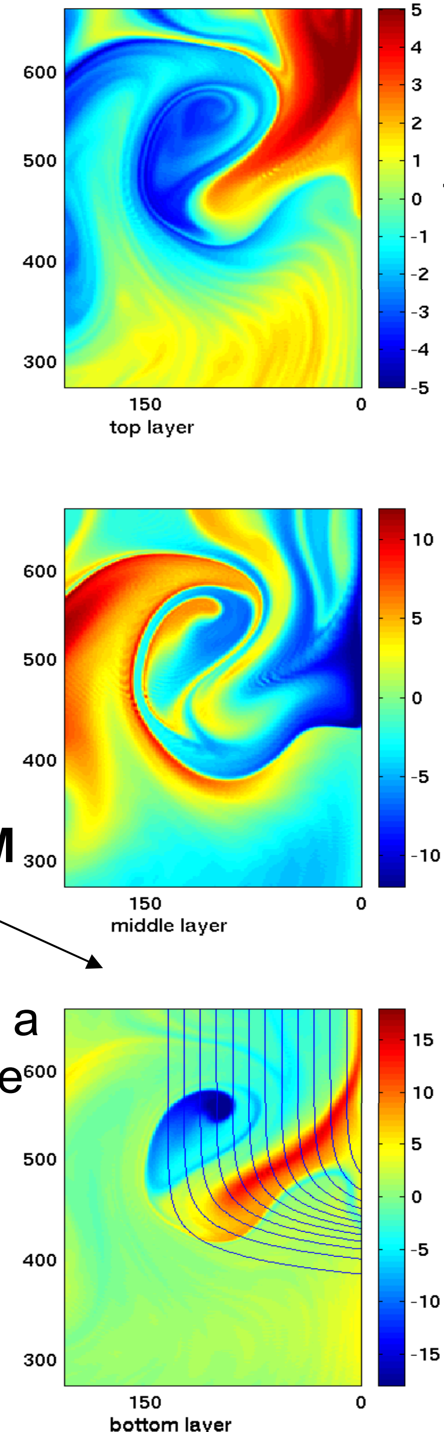
Potential vorticity perturbation:

1) **Vortices form UPSTREAM** from the equilibration of the bottom trapped wave

2) the cyclonic component is immediately destroyed by the shear of the (cyclonic) current

3) the anticyclone moves downstream under the influence of the image at the wall

4) once at the **DOWNSTREAM** step **they detach from the boundary moving towards deeper waters** and often form a dipole ‘grabbing’ water from the boundary current at the downstream step

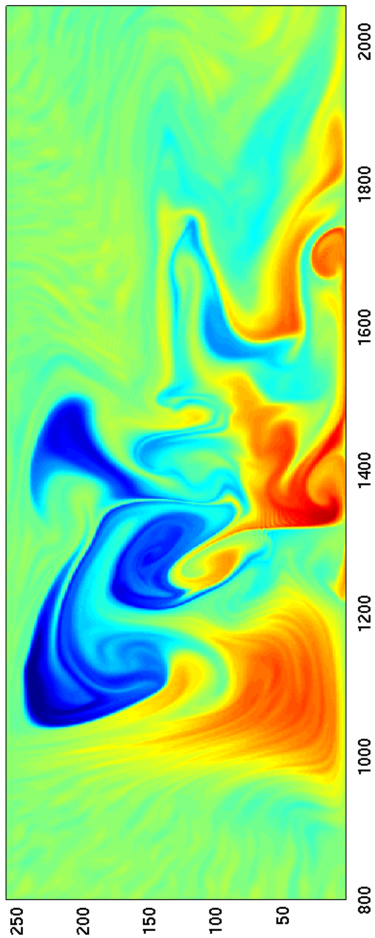


top

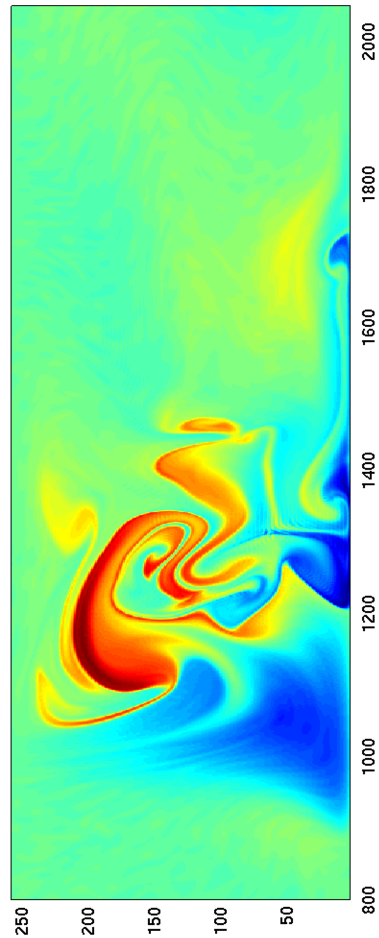
middle

bottom

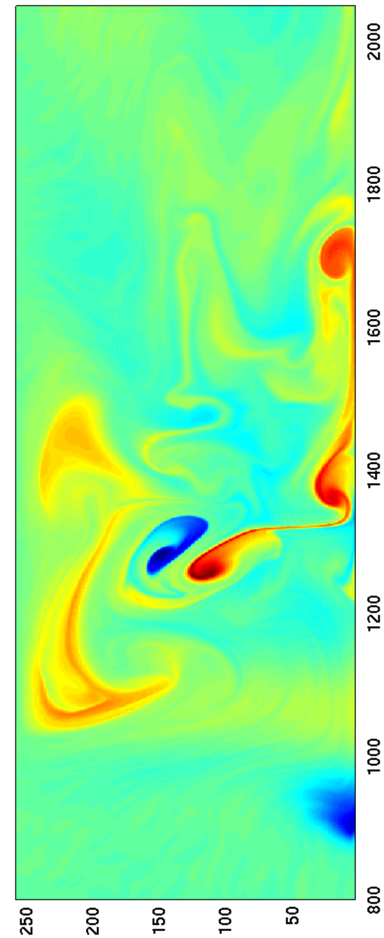
layer 1



layer 2



layer 3

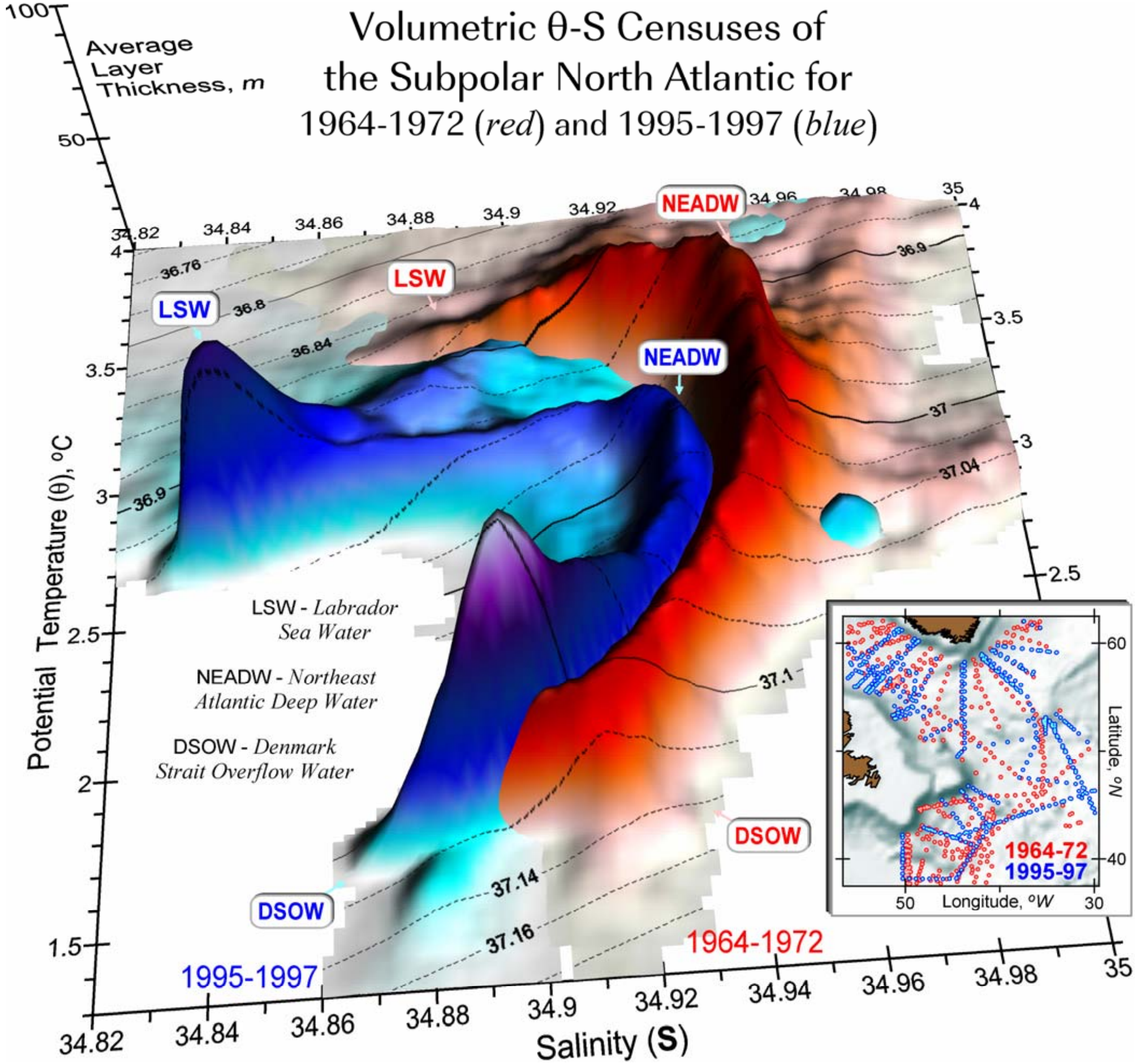


Summary: what we may explain of the Labrador Sea eddy field

- the rate of formation: about 1 every 7 days, but likely seasonally varying. 35% of anticyclones formed at the upstream step end up in the interior. The others are re-absorbed in the current or merge
- the size (R ~ 35 km) and vertical extention of the eddies
- the asymmetry between AC and C

more importantly:

Results suggest that the change in the eddy field seen around 1996 may not be due (only) to a strengthening of the circulation at the surface (NAO?), but could be associated to a strengthening of the bottom current



courtesy of Igor Yashayaev, cover of Progress in Oceanography, 2007