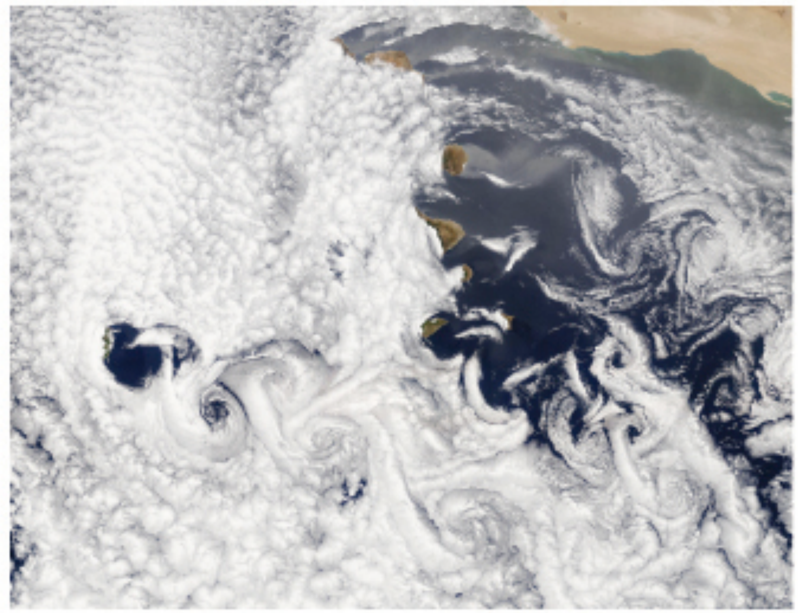
Ocean Modeling - EAS 8803

- The ocean is a *geophysical fluid*
- The goal of ocean modeling is to reproduce numerically the dynamics of the ocean
- Dynamics of the ocean include: mean and time varying circulation, waves, turbulence, instabilities, convection, mixing, jets, etc.
- **Cannot** do ocean modeling without understanding ***geophysical fluid dynamics*** and ***numerical methods***!

Introduction to
Geophysical Fluid Dynamics
Physical and Numerical Aspects



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Academic Press

The Ocean, a geophysical fluid

what scales of motion do we want to model?

large-scale

when considering the large-scale

stratifcation

rotation

- ***Coriolis Force***
- ***Centrifugal Force***

Example of rotation effects

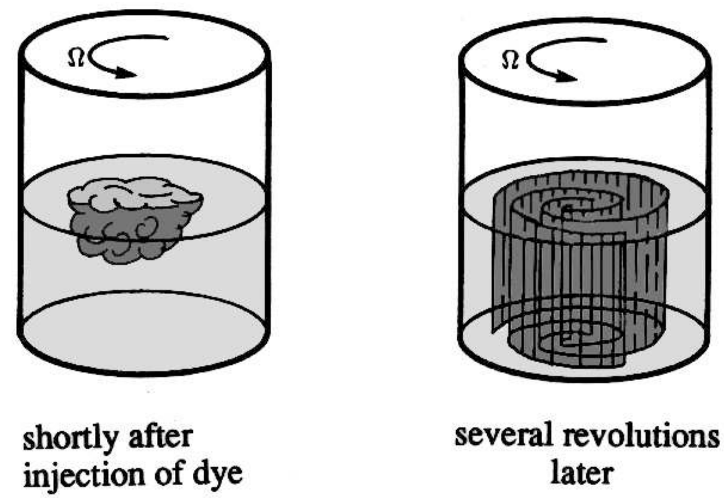


Figure 1-3 Experimental evidence of the rigidity of a rapidly rotating, homogeneous fluid. In a spinning vessel filled with clear water, an initially amorphous cloud of aqueous dye is transformed in the course of several rotations into perfectly vertical sheets, known as *Taylor curtains*.

Scales of Motion

how do we characterize the scales of the ocean?

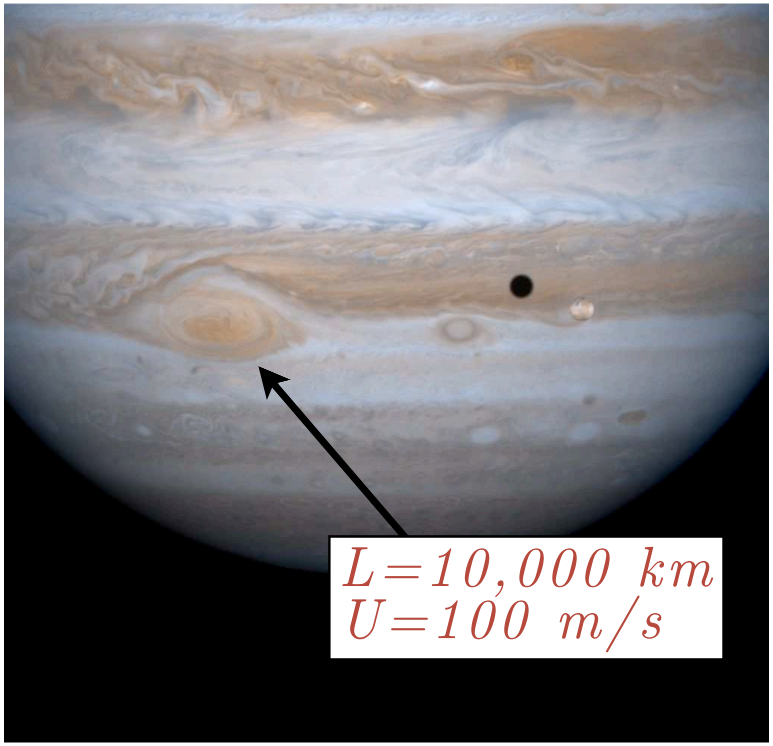
time, length, height, velocity

T L H U

different phenomena are characterized by different scales of motion

useful to ***model only the scales of interest***

Jupiter Red Spot



Scales of Motion

The typical density of the ocean : $\rho_0 = 1025\ \text{kg/m}^3$

However density ρ of the ocean is not uniform, especially in the vertical.

$$\Delta\rho = 1\ \text{kg/m}^3$$

$$\frac{\Delta\rho}{\rho_0} << 1$$

an approximation that we will use to simplify the dynamical equations of motion!

Importance of Rotation

$$\Omega = \frac{2\pi \text{ radians}}{\text{time of one revolution}} = 7.2921 \times 10^{-5} \text{ s}^{-1}.$$

What happens if fluid motion is comparable to the time of one revolution?

$$\omega = \frac{\text{time of one revolution}}{\text{motion time scale}} = \frac{2\pi/\Omega}{T} = \frac{2\pi}{\Omega T}$$

Rotation is important

if

$$\omega \lesssim 1$$

Importance of Rotation

what if $\omega \gtrsim 1$

rotation can still be important, effective timescale $T = L/U$

$$\epsilon = \frac{\text{time of one revolution}}{\text{time taken by particle to cover distance } L \text{ at speed } U} = \frac{2\pi/\Omega}{L/U} = \boxed{\frac{2\pi U}{\Omega L}.}$$

Rossby number

Table 1.1 LENGTH AND VELOCITY SCALES OF MOTIONS IN WHICH ROTATION EFFECTS ARE IMPORTANT

L = 1 m	$U \leq$ 0.012 mm/s
L = 10 m	$U \leq$ 0.12 mm/s
L = 100 m	$U \leq$ 1.2 mm/s
L = 1 km	$U \leq$ 1.2 cm/s
L = 10 km	$U \leq$ 12 cm/s
L = 100 km	$U \leq$ 1.2 m/s
L = 1000 km	$U \leq$ 12 m/s
L = Earth radius = 6371 km	$U \leq$ 74 m/s

Importance of Stratification

when do stratification effect play an important dynamical role?

Kinetic Energy (KE)

$$\sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta \rho g H}$$

Available Potential Energy (APE)

Stratification *is* important

$(\sigma \sim 1)$

$(\sigma \ll 1)$,

Stratification *is not* important

$(\sigma \gg 1)$

Importance of Stratification and Rotation

what happens when both rotation and stratification are important?

$$\epsilon \sim 1 \text{ and } \sigma \sim 1$$

$$\epsilon = \frac{2\pi U}{\Omega L} \qquad \sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta\rho g H}$$

$$L \sim \frac{U}{\Omega} \quad \text{and} \quad U \sim \sqrt{\frac{\Delta\rho}{\rho_0} g H}$$

$$\boxed{L \sim \frac{1}{\Omega}\sqrt{\frac{\Delta\rho}{\rho_0} g H}}$$

Rossby deformation Radius

Length scale over which motions take place

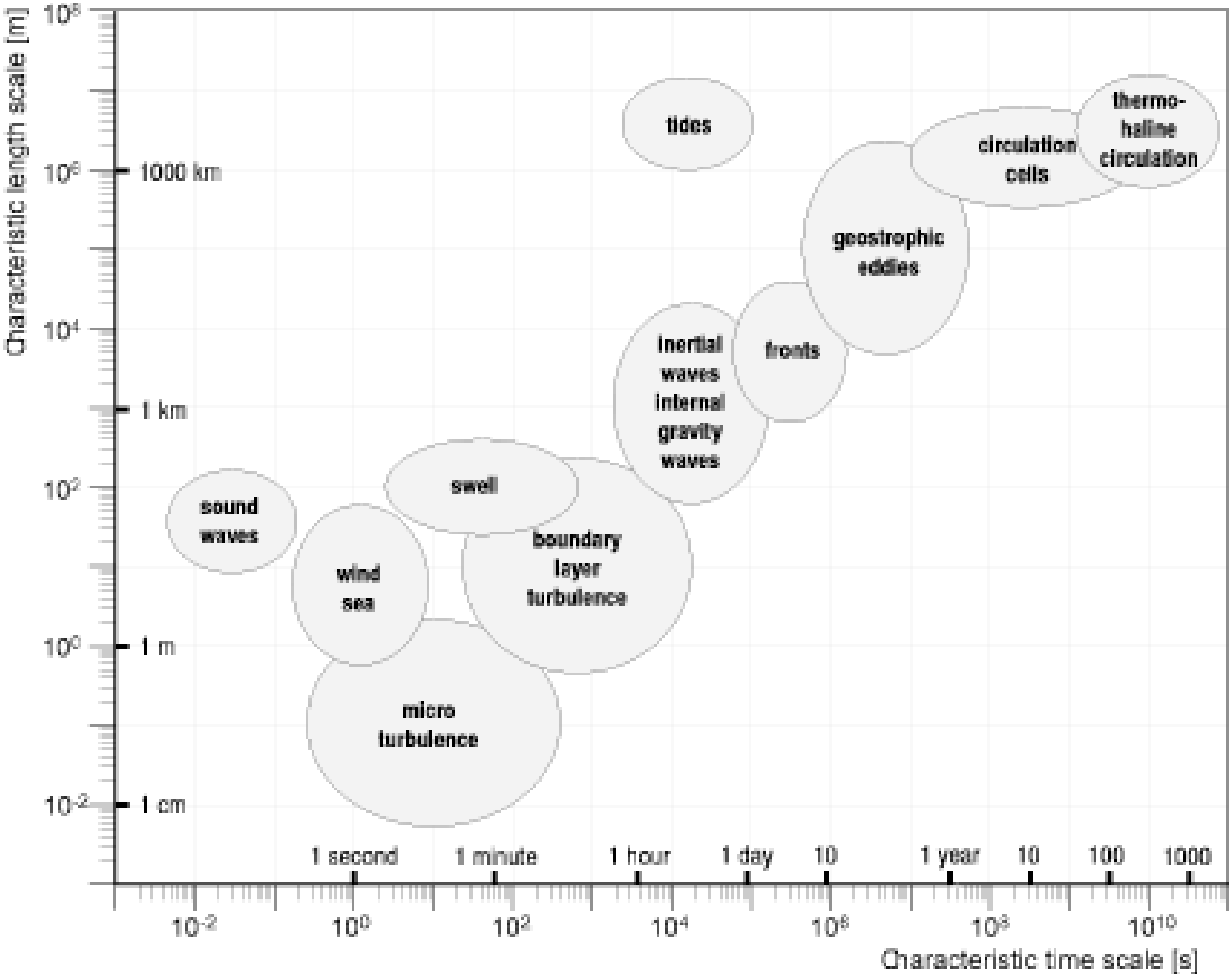
$$L_{\text{atmosphere}} \sim 500 \text{ km} \qquad U_{\text{atmosphere}} \sim 30 \text{ m/s}$$
$$L_{\text{ocean}} \sim 60 \text{ km} \qquad U_{\text{ocean}} \sim 4 \text{ m/s}$$

Typical Scales of Ocean and Atmosphere Phenomena

Table 1.2 LENGTH, VELOCITY AND TIME SCALES IN THE EARTH'S ATMOSPHERE AND OCEANS

Phenomenon	Length Scale L	Velocity Scale U	Time Scale T
Atmosphere:			
Microturbulence	10–100 cm	5–50 cm/s	few seconds
Thunderstorms	few km	1–10 m/s	few hours
Sea breeze	5–50 km	1–10 m/s	6 hours
Tornado	10–500 m	30–100 m/s	10–60 minutes
Hurricane	300–500 km	30–60 m/s	Days to weeks
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
Microturbulence	1–100 cm	1–10 cm/s	10–100 s
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Tides	Basin scale	1–100 m/s	Hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Fronts	1–20 km	0.5–5 m/s	Few days
Eddies	5–100 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Typical Scales of Ocean Processes



Modeling the Ocean and the Atmosphere

example of an early computation (1928)

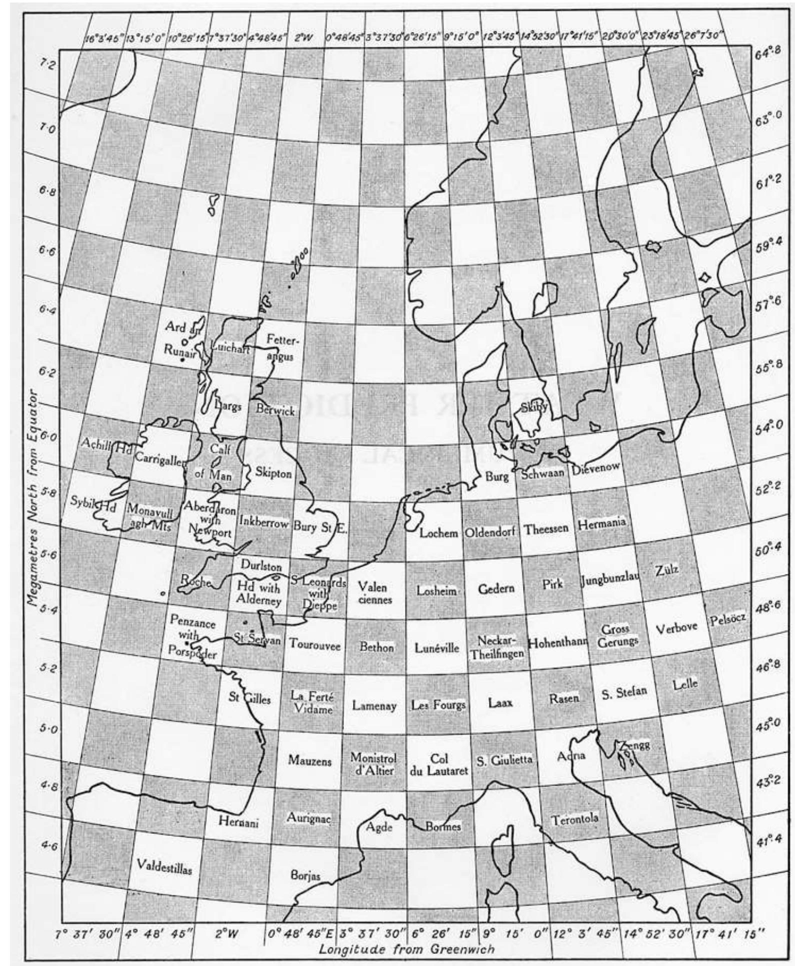
Complex differential equations

↓

Set of arithmetic operations

↓

step by step method of solution (*model time-stepping*) at selected points in space (*model spatial grid*)



Model grid used by Lewis Fry Richardson

CAVEAT ! The concept of numerical stability was not known until 1928 when it was elucidated by Richard Courant, Karl Friedrichs and Hans Lewy.

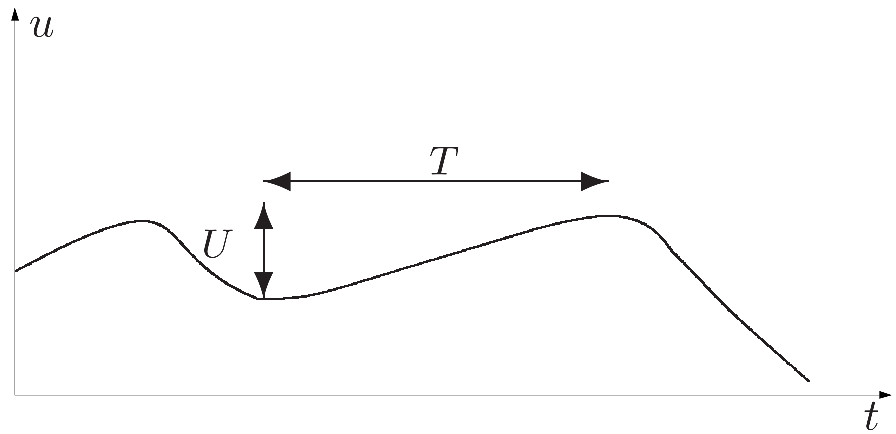


Figure 1-11 Time-scale analysis of a variable u. The time scale T is the time interval over which the variable u exhibits variations comparable to its standard deviation U.

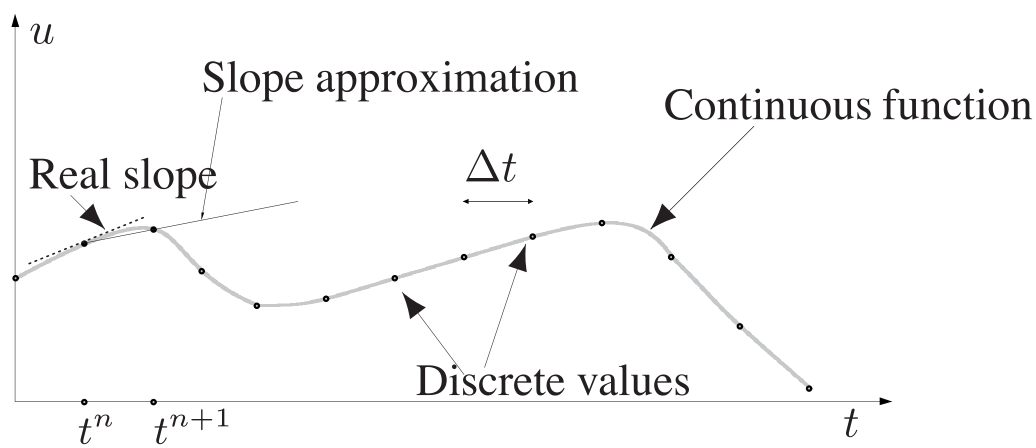


Figure 1-12 Representation of a function by a finite number of sampled values and approximation of a first derivative by a finite difference over Δt.

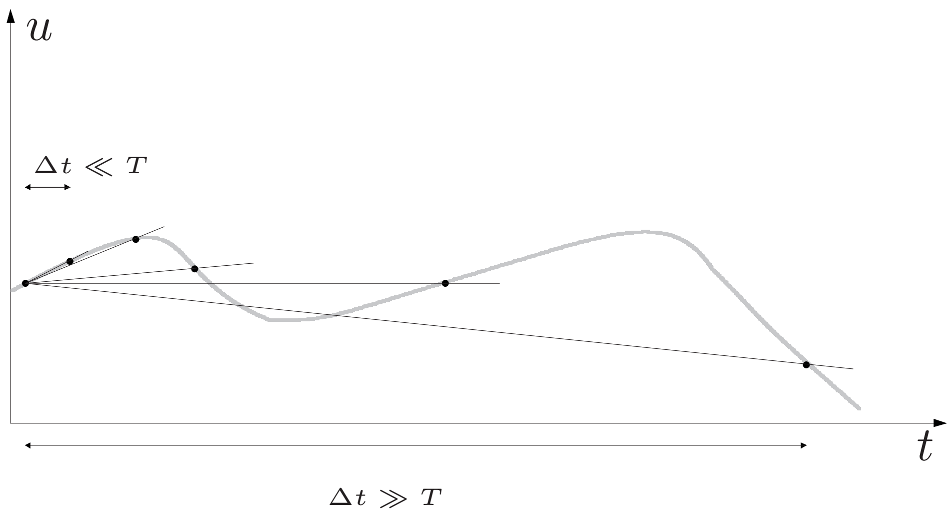
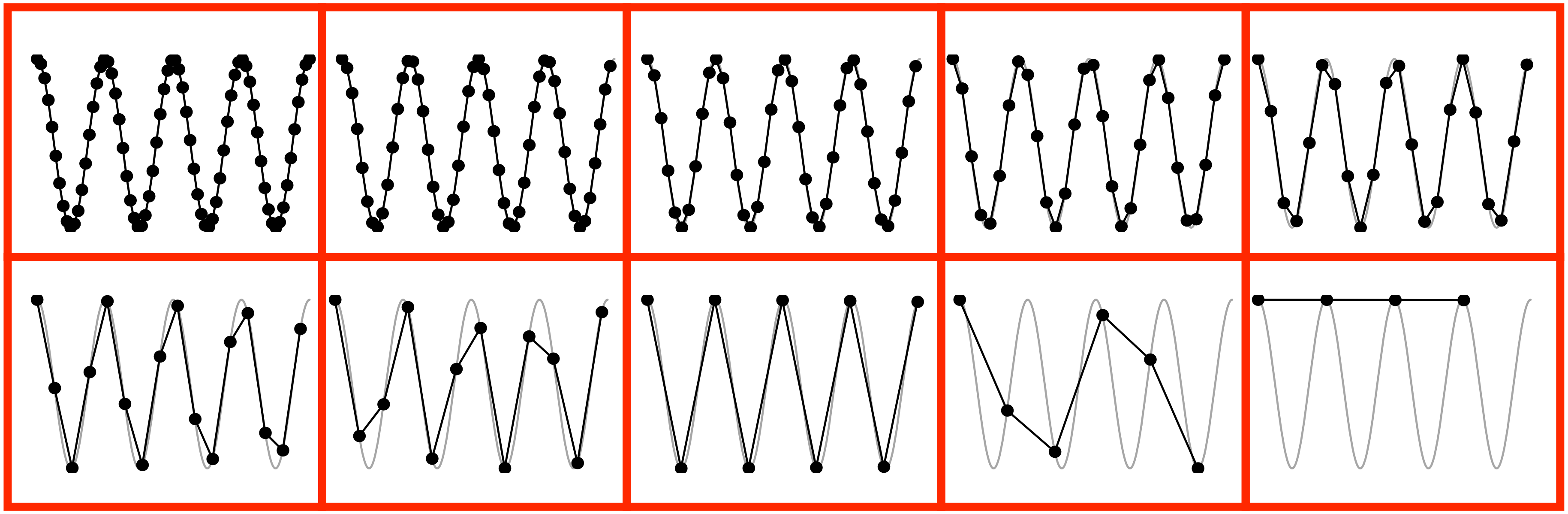


Figure 1-13 Finite differencing with various Δt values. Only when the time step is sufficiently short compared to the time scale, $\Delta t \ll T$, is the finite-difference slope close to the derivative, *i.e.*, the true slope.

Δt *increases* →



Δt *increases* →