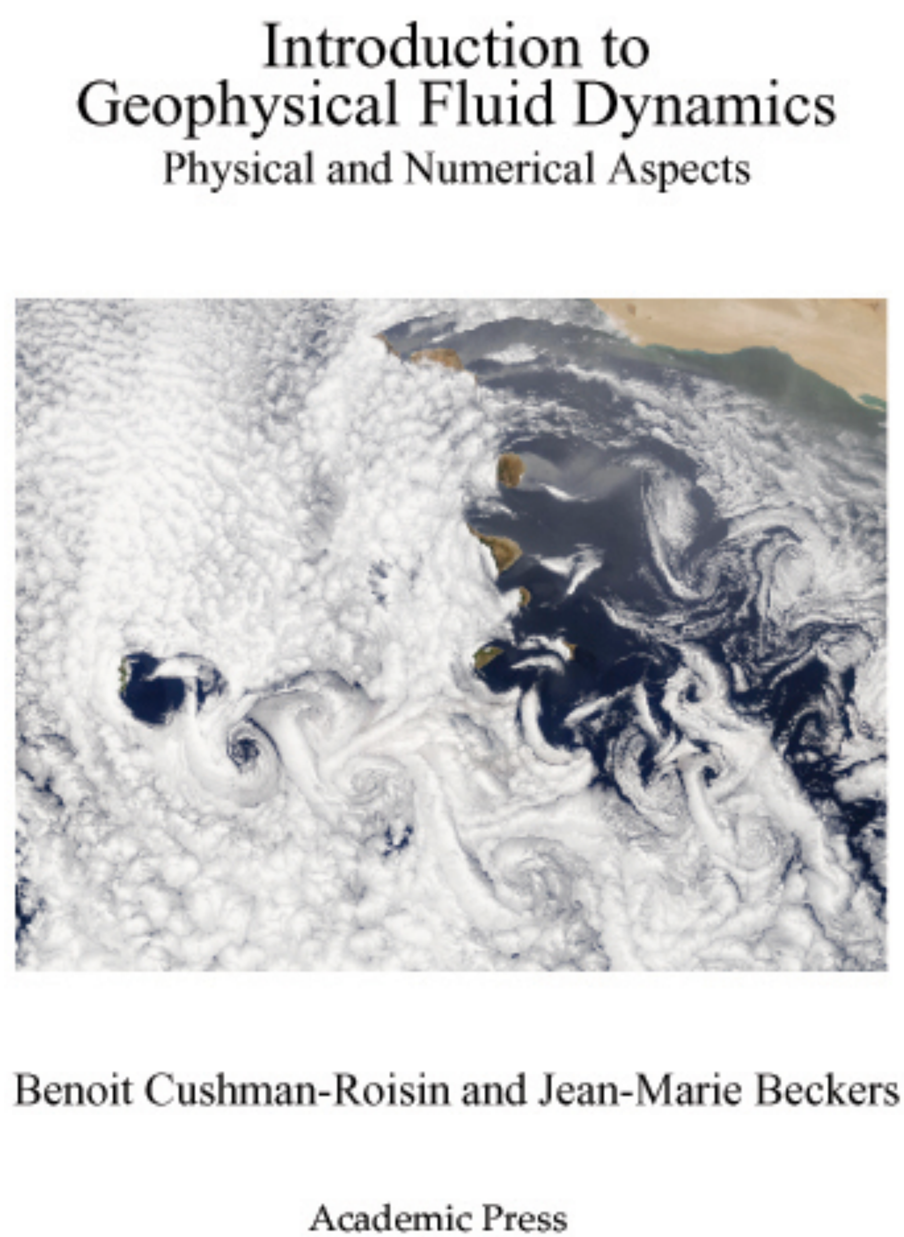
Ocean Modeling - EAS 8803

- The ocean is a *geophysical fluid*

- The goal of ocean modeling is to reproduce numerically the dynamics of the ocean

- Dynamics of the ocean include: mean and time varying circulation, waves, turbulence, instabilities, convection, mixing, jets, etc.

- **Cannot** do ocean modeling without understanding ***geophysical fluid dynamics*** and ***numerical methods***!



The Ocean, a geophysical fluid

what scales of motion do we want to model?

large-scale

when considering the large-scale

stratifcation

rotation

- *Coriolis Force*
- *Centrifugal Force*

Example of rotation effects

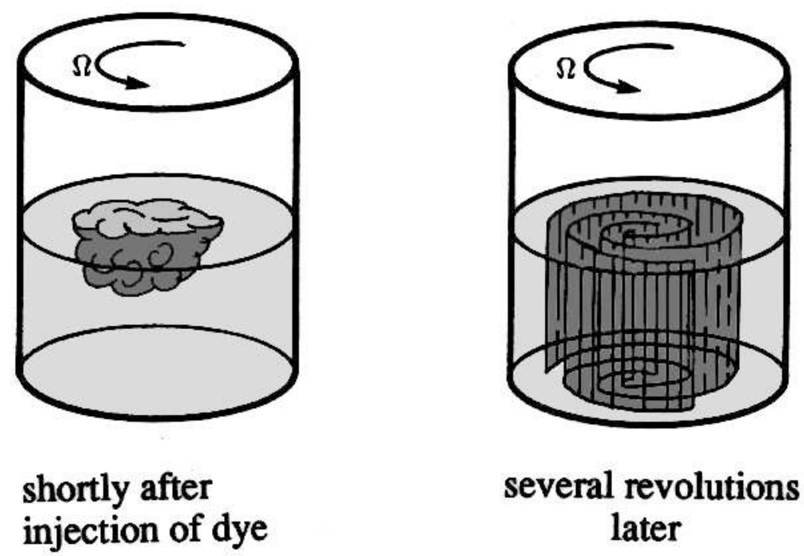


Figure 1-3 Experimental evidence of the rigidity of a rapidly rotating, homogeneous fluid. In a spinning vessel filled with clear water, an initially amorphous cloud of aqueous dye is transformed in the course of several rotations into perfectly vertical sheets, known as *Taylor curtains*.

Scales of Motion

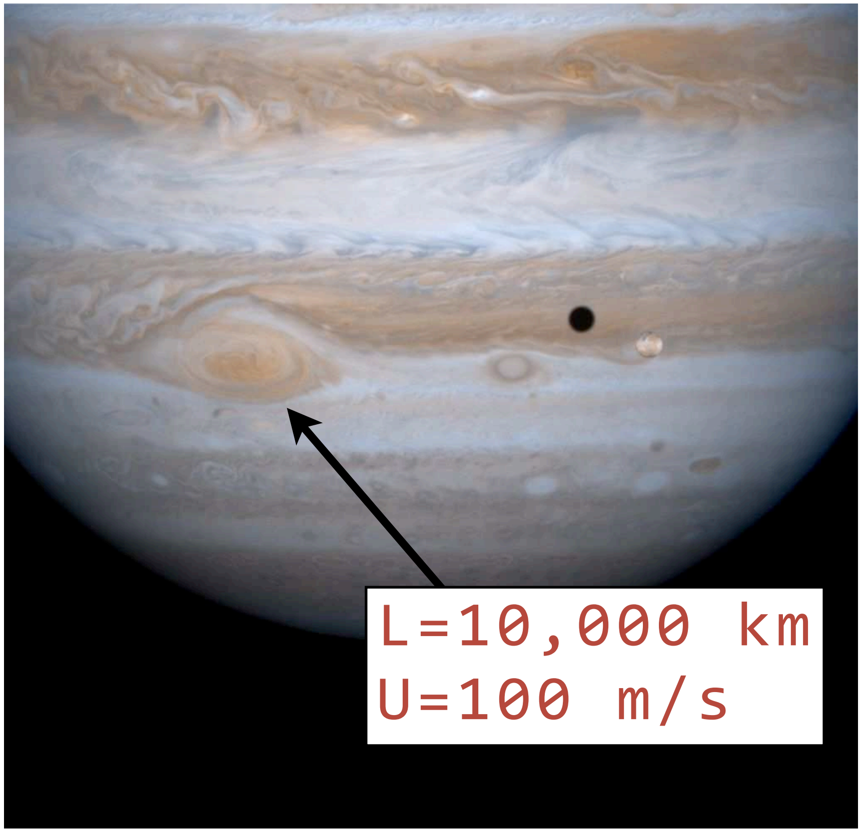
how do we characterize the scales of the ocean?

time, length, height, velocity

T L H

U

Jupiter Red Spot



different phenomena are characterized by different scales of motion

useful to ***model only the scales of interest***

Scales of Motion

The typical density of the ocean : $\rho_0 = 1025 \text{ kg/m}^3$

However density ρ of the ocean is not uniform, especially in the vertical.

$$\Delta\rho = 1 \text{ kg/m}^3$$

$$\frac{\Delta\rho}{\rho_0} << 1$$

an approximation that we will use to simplify the dynamical equations of motion!

Importance of Rotation

$$\Omega = \frac{2\pi \text{ radians}}{\text{time of one revolution}} = 7.2921 \times 10^{-5} \text{ s}^{-1}.$$

What happens if fluid motion is comparable to the time of one revolution?

$$\omega = \frac{\text{time of one revolution}}{\text{motion time scale}} = \frac{2\pi/\Omega}{T} = \frac{2\pi}{\Omega T}$$

Rotation is important

if

$$\omega \lesssim 1$$

Importance of Rotation

what if $\omega \gtrsim 1$

rotation can still be important, effective timescale $T = L/U$

$$\begin{aligned}\epsilon &= \frac{\text{time of one revolution}}{\text{time taken by particle to cover distance } L \text{ at speed } U} \\ &= \frac{2\pi/\Omega}{L/U} = \boxed{\frac{2\pi U}{\Omega L}.}\end{aligned}$$

Rossby number

Table 1.1 LENGTH AND VELOCITY SCALES OF MOTIONS IN WHICH ROTATION EFFECTS ARE IMPORTANT

L = 1 m	$U \leq$ 0.012 mm/s
L = 10 m	$U \leq$ 0.12 mm/s
L = 100 m	$U \leq$ 1.2 mm/s
L = 1 km	$U \leq$ 1.2 cm/s
L = 10 km	$U \leq$ 12 cm/s
L = 100 km	$U \leq$ 1.2 m/s
L = 1000 km	$U \leq$ 12 m/s
L = Earth radius = 6371 km	$U \leq$ 74 m/s

Importance of Stratification

when do stratification effect play an important dynamical role?

Kinetic Energy (KE)

$$\sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta \rho g H}$$

Available Potential Energy (APE)

Stratification *is* important

$(\sigma \sim 1)$

$(\sigma \ll 1)$

Stratification *is not* important

$(\sigma \gg 1)$

Importance of Stratification and Rotation

what happens when both rotation and stratification are important?

$$\epsilon \sim 1 \text{ and } \sigma \sim 1$$

$$\epsilon = \frac{2\pi U}{\Omega L} \qquad \sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta\rho g H}$$

$$L \sim \frac{U}{\Omega} \quad \text{and} \quad U \sim \sqrt{\frac{\Delta\rho}{\rho_0} g H}$$

$$\boxed{L \sim \frac{1}{\Omega}\sqrt{\frac{\Delta\rho}{\rho_0} g H}}$$

Rossby deformation Radius

Length scale over which motions take place

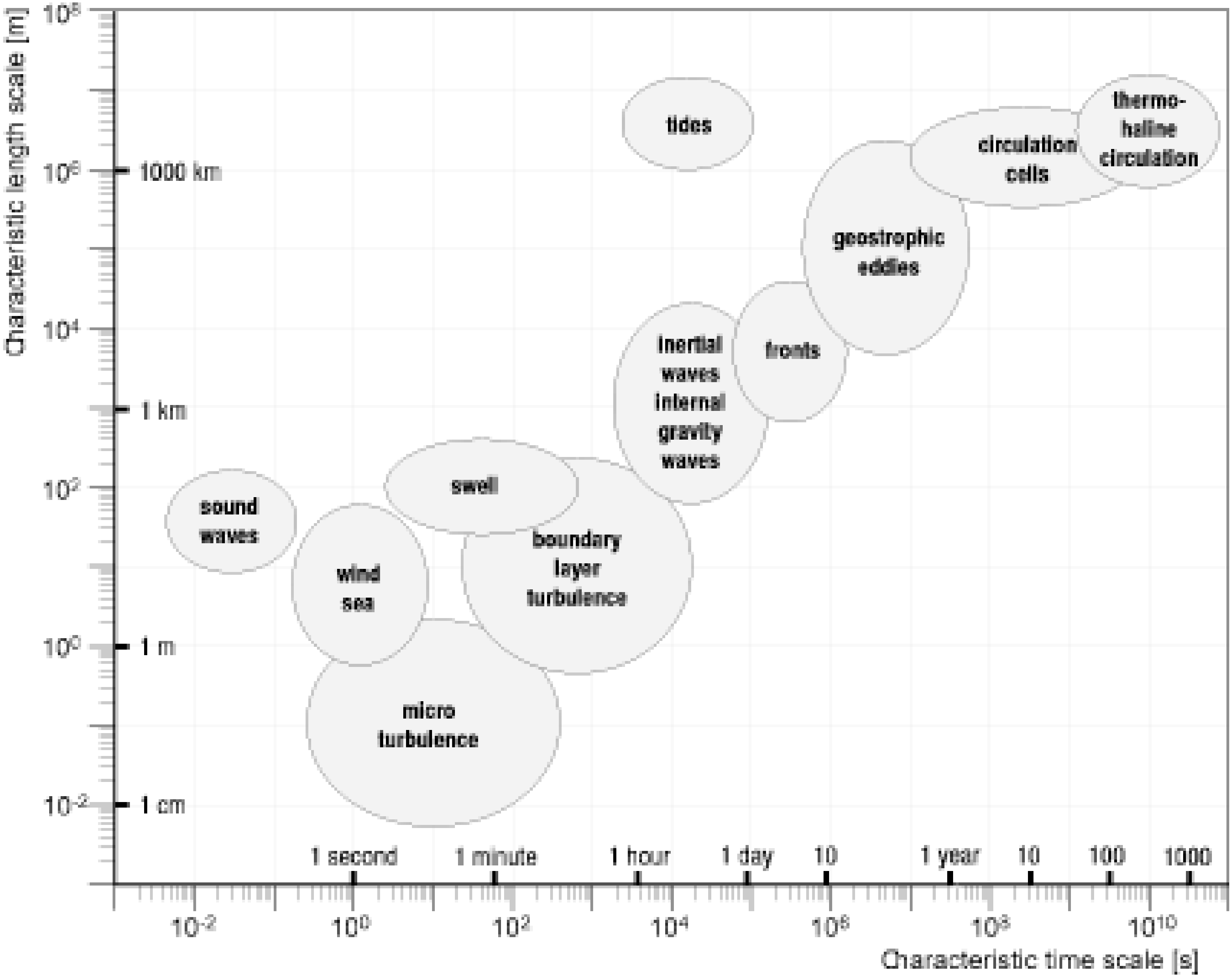
$$\begin{array}{llll} L_{\text{atmosphere}} & \sim 500\ \text{km} & U_{\text{atmosphere}} & \sim 30\ \text{m/s} \\ L_{\text{ocean}} & \sim \ \ 60\ \text{km} & U_{\text{ocean}} & \sim \ \ 4\ \text{m/s} \end{array}$$

Typical Scales of Ocean and Atmosphere Phenomena

Table 1.2 Length, velocity and time scales in the earth's atmosphere and oceans

Phenomenon	Length Scale L	Velocity Scale U	Time Scale T
Atmosphere:			
Microturbulence	10–100 cm	5–50 cm/s	few seconds
Thunderstorms	few km	1–10 m/s	few hours
Sea breeze	5–50 km	1–10 m/s	6 hours
Tornado	10–500 m	30–100 m/s	10–60 minutes
Hurricane	300–500 km	30–60 m/s	Days to weeks
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
Microturbulence	1–100 cm	1–10 cm/s	10–100 s
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Tides	Basin scale	1–100 m/s	Hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Fronts	1–20 km	0.5–5 m/s	Few days
Eddies	5–100 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Typical Scales of Ocean Processes



Modeling the Ocean and the Atmosphere

example of an early computation (1928)

Complex differential equations

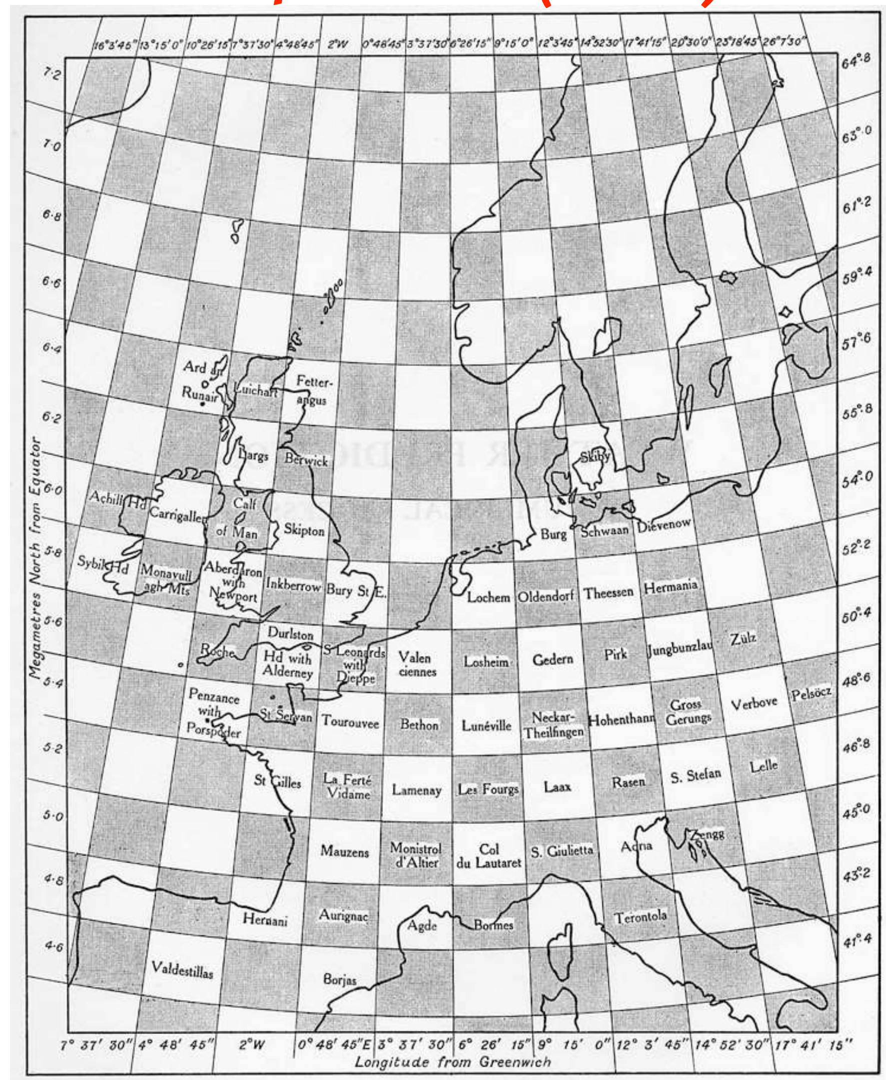
Set of arithmetic operations

step by step method of solution

(*model time-stepping*)

at selected points in space

(*model spatial grid*)



Model grid used by Lewis Fry Richardson

CAVEAT ! The concept of numerical stability was not known until 1928 when it was elucidated by Richard Courant, Karl Friedrichs and Hans Lewy.

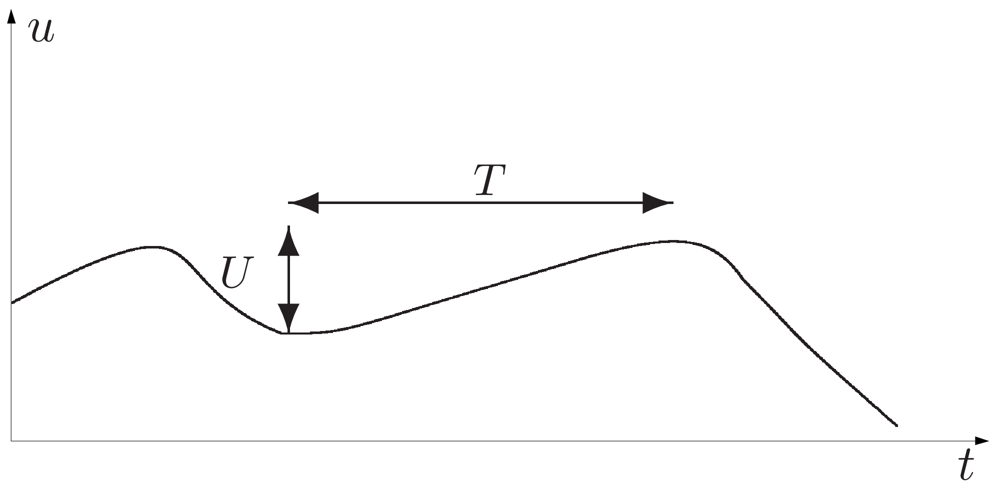


Figure 1-11 Time-scale analysis of a variable u. The time scale T is the time interval over which the variable u exhibits variations comparable to its standard deviation U.

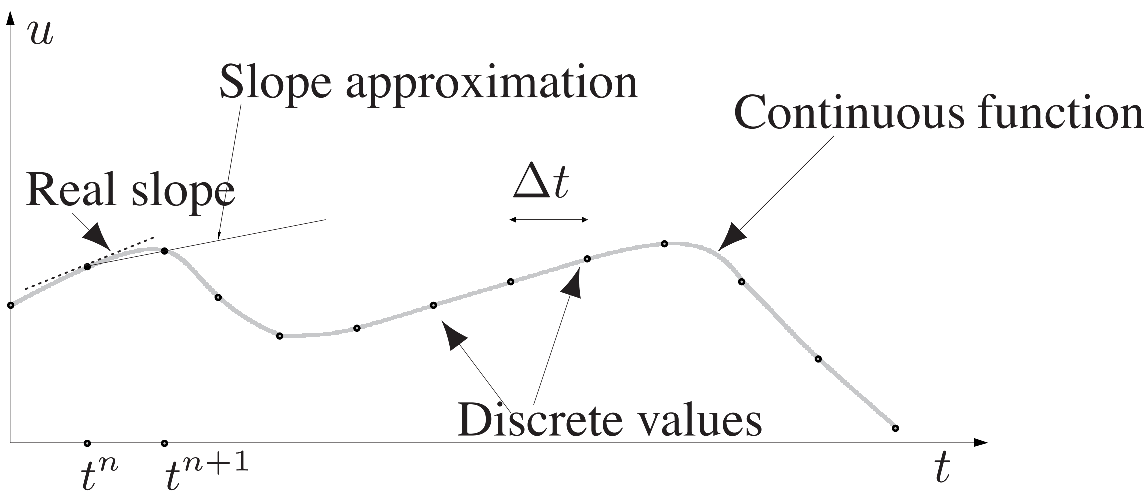


Figure 1-12 Representation of a function by a finite number of sampled values and approximation of a first derivative by a finite difference over Δt.

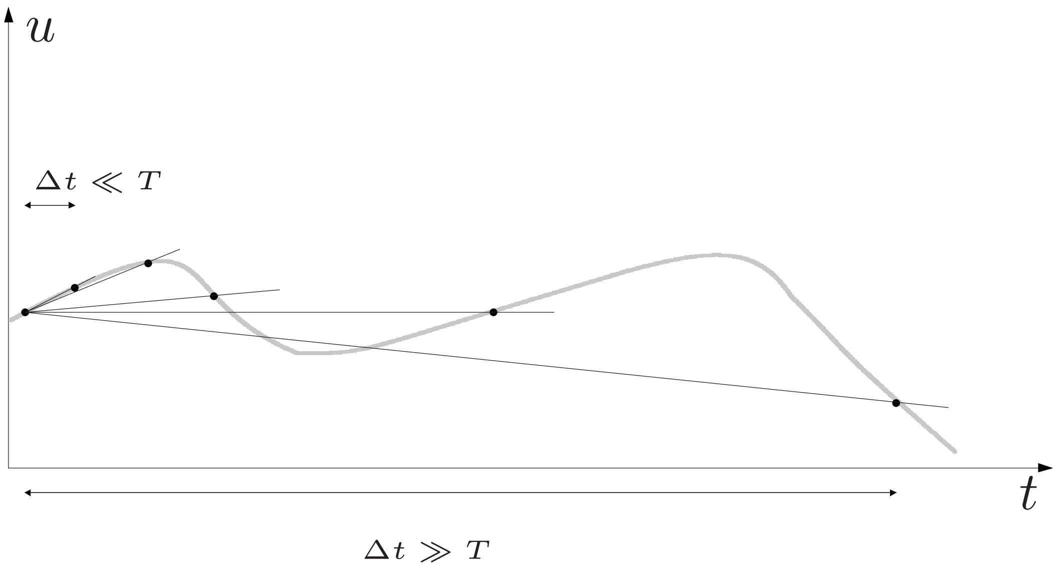
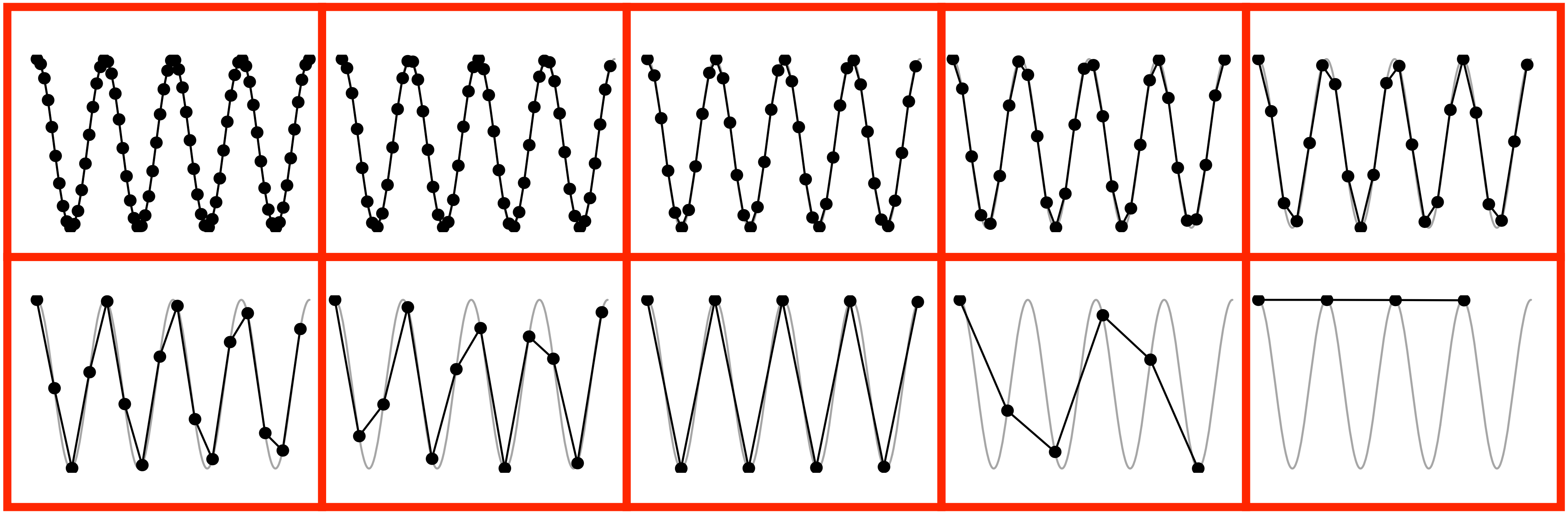


Figure 1-13 Finite differencing with various Δt values. Only when the time step is sufficiently short compared to the time scale, $\Delta t \ll T$, is the finite-difference slope close to the derivative, *i.e.*, the true slope.

Δt *increases* →



Δt *increases* →