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DYNAMICS OF STEADY OCEAN CURRENTS IN THE LIGHT OF EXPERIMENTAL FLUID MECHANICS

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INTRODUCTION

The present investigation may be regarded as a part of a systematic effort to introduce into meteorology and physical oceanography methods and results which for a number of years have contributed to the rapid growth and increasing practical significance of experimental fluid mechanics. This science has recognized that the exact character of the forces controlling the motion of a turbulent fluid is not known and that consequently there is very little justification for a purely theoretical attack on problems of a practical character. For this reason fluid mechanics has been forced to develop a research technique all of its own, in which the theory is developed on the basis of experiments and then used to predict the behavior of fluids in cases which are not accessible to experimentation.

In oceanography it has long been regarded as an axiom that the movements of the water are controlled by three forces, the horizontal pressure gradient, the deflecting force, and the frictional force resulting from the relative motion of superimposed strata. It is significant that thirty-five years of intensive theoretical work on this basis have failed to produce a theory capable of explaining the major features of the observed oceanic circulation below the pure drift current layer.

The present investigation considers a force which has been completely disregarded by theoretical investigators although its existence has been admitted implicitly by practically everyone who has approached physical oceanography from the descriptive side, namely the frictional force resulting from large-scale horizontal mixing. The introduction of this force permits us to see how motion generated in the surface layers may be diffused and finally dissipated without recourse to doubtful frictional forces at the bottom of the ocean.

A great number of practical hydrodynamic investigations of the observed oceanic current systems consist mainly in velocity calculations with the aid of the circulation theorem. Without denying the great practical value of the circulation theorem, the present investigation endeavors to emphasize a fact which by this time should have been generally accepted but which it not always kept in mind, namely the impossibility of drawing any conclusions regarding the cause of oceanic motions from the ordinary routine application of the circulation theorem.

In the first part of the paper the principal imperfections of the present theory for the oceanic circulation are set forth. Frictional forces due to horizontal mixing are then introduced and the effect of the earth's rotation on the horizontal eddy velocities analyzed. Tollmien's theory for the mixing along the edges of a steady stream moving through a resting fluid is then discussed and certain experimental verifications are described. With the aid of a principle first stated by G. I. Taylor, Tollmien's results are applied to current systems subject to a deflecting force. Finally certain important modifications resulting from the stratification in the ocean are treated.

In the second part of the paper an attempt is made to trace the mixing between the Gulf Stream and its surroundings with the aid of the observed distribution of temperature, salinity and oxygen. The results of this qualitative analysis seem to bear out the theoretical predictions.

The theory set forth is utterly incomplete, and serious objections may be raised against the looseness of the reasoning on which it is based. Nevertheless, the author believes that it may serve as a useful working hypothesis, since its predictions refer to an idealized stratified ocean and not to a non-existent homogeneous medium.

The present paper is to a large extent the result of fruitful cooperation between a number of persons. Dr. H. Peters of the Massachusetts Institute of Technology not only carried out certain experimental tests of Tollmien's theory but also, in a number of discussions, directed my attention to various investigations bearing on the relative merits of the momentum transfer and the vorticity transfer theories.

Mr. C. O'D. Iselin of the Woods Hole Oceanographic Institution has contributed his vast knowledge of the hydrographic conditions in the North Atlantic. Without his active cooperation it would have been impossible to carry through the investigation to a point where it could be tested against observations. Several of the conclusions here derived from purely theoretical considerations have already been reached by Mr. Iselin from a study of the hydrographic data collected by the Woods Hole Oceanographic Institution.

Mr. H. R. Seiwell's investigations of the oxygen distribution in the North Atlantic have been particularly helpful and are responsible for the choice of oxygen as an indicator of horizontal mixing.

The author is indebted to Dr. H. B. Bigelow for various helpful suggestions.

A brief account of the principal theoretical results presented below was given before the annual meeting of the Institute of the Aeronautical Sciences in New York, January, 1936.

At the date of writing this introduction, Dr. A. E. Parr of Yale University informs me that he has been led to conclusions of substantially the same nature as some of the ones here presented, through a study of recent hydrographic data from the Caribbean. Dr. Parr's results will be published in *Journal du Conseil.*

Woods Hole, July 15, 1936

I. THEORETICAL DISCUSSION

A. Formulation of Problem

Anybody who has attempted to construct, for his own satisfaction, theoretical working models of the permanent current system in the ocean or in the air or of some of the apparently steady phenomena of the secondary circulation, sooner or later runs up against the apparent impossibility of finding forces capable of producing, in the interior of the media under consideration, horizontal convergence or divergence on a scale comparable to that which actually must occur in nature. In cyclonic regions, surface friction produces an easily observed transport of air across the isobars towards lower pressure. Since the gradient wind supposed to prevail at higher levels is very nearly free from divergence there is apparently no way in which the accumulating surface air may be removed. One would therefore expect a rapid decay or filling up of these low pressure systems. Nevertheless, particularly the occluded cyclones of higher latitudes and the hurricanes of lower latitudes often seem to be characterized by a condition of approximate dynamic equilibrium.

A similar problem appears in the interpretation of the horizontal circulation of the ocean. The permanent anticyclonic wind system of the North Atlantic Ocean produces a steady accumulation of surface water in lower latitudes and a corresponding slope of the sea surface. The resulting gradient current system should be very nearly free from horizontal divergence and thus incapable of re-establishing equilibrium. To avoid this difficulty Ekman,[1] in his general theory of the circulation of a homogeneous ocean, assumes that the bottom friction is so strong that it produces a divergence sufficient to offset the wind-produced surface convergence. It is easily shown that the bottom friction required for this purpose must be of the same order of magnitude as the surface friction.

Ekman's solution implies that bottom water and surface water are equivalent. In each region of surface convergence and bottom divergence there must be a descending motion, in each region of surface divergence and bottom convergence there must be an ascending motion, so that bottom and surface water continually replace each other. While this may be acceptable in the ideal case of a homogeneous ocean, it is in sharp disagreement with observed conditions in the real, stratified ocean.

According to Ekman's theory the equatorial side of the subtropical Highs must be characterized by such accumulation of surface water. The observed steep thermocline in these regions shows that the accumulation and sinking of surface water must cease within a depth of a few hundred meters, in contrast with the theoretical prediction, although there are definite indications that strong horizontal convergence and sinking must occur in these upper layers.

Since the vertical circulation does not extend all the way down it may be argued that the water, because of its stratification, has a cellular structure, each cell being separated through approximately horizontal surfaces of discontinuity from the cells above and below. Each boundary surface would then act as a "false" bottom and each cell would have a practically independent circulation. In order to have steady conditions and zero horizontal divergence in each cell, it would be necessary for the shearing stresses at each boundary to be of the same order of magnitude as those at the surface. This stress distribution would produce a much stronger circulation in the bottom cell than indicated by available data. Furthermore, for each new cell introduced the surface current velocity is raised, so that the suggested scheme most likely would produce impossible surface velocities.

It is no doubt possible to overcome some of these difficulties locally by considering the deviations from gradient flow associated with inertia forces. This is the line of attack followed by Ekman in his latest investigations. It is as yet impossible to estimate completely the extent to which this much needed extension of the theory will eliminate the difficulties listed above. However, in this connection the following comment is pertinent:

The horizontal circulation of the southern half of the North Atlantic may be represented as a gigantic *stationary* anticyclonic eddy maintained by the permanent anticyclonic wind system over the same area. Since the mean motion is steady, the mean total torque round a vertical axis must vanish. In Ekman's theory this is accomplished through the introduction of frictional forces at the bottom, the torque of which balances the wind torque. The consideration of inertia forces in no way removes the need for this balancing frictional force at the bottom. Actually observations indicate that the motion near the bottom is vanishingly small and thus incapable of producing frictional forces of any significance.

An inspection of a current chart for the North Atlantic indicates that strong eddying motion occurs at many places along the borders of the basin. *Thus it seems possible that the required balance may be established through frictional forces originating on the continental slopes and transmitted through the water as shearing stresses acting on vertical surfaces parallel to the horizontal current components.* For the sake of brevity shearing stresses of this type will here be referred to as lateral stresses, while the designation normal stresses is reserved for stresses acting on horizontal surfaces and produced by the vertical variation in horizontal velocity. It is evident, from a study of the relative horizontal and vertical dimensions of atmospheric and oceanic systems, that the lateral stresses must be many times larger than the normal ones if they are to be of any dynamic significance.

The idea that momentum may be transferred horizontally through turbulence is not new. In a much-discussed paper published in 1921 Defant[2] assumed that the travelling cyclones and anticyclones may be regarded as turbulent elements superimposed on the mean circulation of the atmosphere in middle latitudes. Defant used this conception of the general circulation to compute the advective transfer of heat from the equator to the poles. However, in Defant's case the eddying components are quite large compared to the mean motion, so large, in fact, that the mean motion of the air is often completely obscured by the presence of the eddying motion. It is doubtful that these large eddies derive their energy from the mean motion, and perhaps more likely that the reverse is true. Thus it appears desirable to select for study a steady fluid system characterized by a well-established primary mean motion and to determine the rôle played by lateral shearing stresses in the dynamics of this system.

Richardson and Proctor[3] have investigated horizontal diffusion in atmospheric currents by means of the scattering of volcanic ash and the scattering of small toy balloons. For distances ranging between 3 km. and 86 km. these authors obtained values of the horizontal diffusivity varying between $2 \cdot 10^6$ and $1.3 \cdot 10^9$ cm.^{2}/sec. It is reasonable to assume that the turbulent mechanism responsible for this scattering must produce an equally intensive lateral diffusion of horizontal momentum. The lateral stresses introduced above are simply a measure of this lateral eddy transport of momentum.

If Richardson's and Proctor's coefficients are expressed as eddy-viscosities, they range from $2.5 \cdot 10^3$ to $1.6 \cdot 10^6$ grams/cm.sec. Thus they are intermediate in magnitude between the values obtained from the study of vertical wind gradients, 10^2 grams/cm.sec., and the values obtained by Defant from an analysis of the general circulation as a turbulent process, 10^8 grams/cm.sec. In Defant's analysis of the general circulation the individual turbulent elements are supposed to consist of travelling cyclones and anticyclones or, more properly, of large bodies of air from different source regions. The diffusion process measured by Richardson and Proctor, and studied from another point of view in the present paper, deals with phenomena within a single air or ocean current and along its boundaries. It presupposes the existence of eddies whose dimensions must be measured in fractions of a kilometer up to, perhaps, twenty or thirty kilometers. The remarkable uniformity in air mass characteristics so often observed in our aerological data suggests that horizontal diffusion on such a large scale must occur with great regularity in the atmosphere. It is rather surprising then to find that the dynamic consequences of this horizontal diffusion mechanism never have been investigated.

Before proceeding, it may be worth while to point out how lateral shearing stresses affect the horizontal divergence. On the northern hemisphere, steady, non-accelerated motion in the atmosphere or in the ocean is characterized by the fact that to a given horizontal force P there corresponds a horizontal momentum M directed 90° to the right from P and having the value

$$M = \frac{P}{2\omega \sin L}, \tag{1}$$

where L is the latitude and ω is the angular velocity of the earth. As an illustration, consider a vertical air column in a field of straight, parallel isobars. This column is acted upon by the horizontal pressure gradient and by the frictional force between the air and the ground. It is evident that the component of its momentum across the isobars must correspond to the component of ground friction parallel to the isobars. If the same column of air is subject not only to normal stresses but also to suitable lateral shearing stresses, the resultant force along the isobar direction and thus also the total flow across the isobars may be made to vanish.

Because of the earth's rotation, the effect of the normal shearing stresses originating at a horizontal boundary vanish within a relatively short vertical distance. Outside these shallow boundary layers the velocities vary only slowly along the vertical, at least when there is steady motion and when the medium considered is in barotropic equilibrium; thus we are permitted to assume that the lateral shearing stresses are reasonably independent of the vertical coordinate through fairly deep strata. This effect of the earth's rotation simplifies a separation of the effects of lateral and normal stresses; such a separation, on the other hand, is not readily possible in the case of small-scale hydraulic experiments.

The balance of forces in a horizontal direction normal to the mean motion, which consists in an equilibrium between deflecting force and horizontal pressure gradient, is not materially affected by the presence of lateral stresses. *This balance, which for the atmosphere takes the form of the ordinary gradient wind relationship and which is also utilized for so called "dynamic velocity calculations" of ocean currents, does not prescribe a definite velocity profile across the current.* More specifically, if we consider the mass distribution in a certain vertical plane, it is always possible to find a distribution of velocities normal to this plane such that the resulting deflecting force everywhere balances the horizontal pressure gradient resulting from the mass distribution (distribution of solenoids). Conversely, it is always possible to find a mass distribution in a vertical plane such that the resulting horizontal pressure gradient balances the deflecting force associated with an arbitrary distribution of velocities normal to the plane.

On the other hand, the effect of lateral stresses acting in the direction of the motion must be to produce certain characteristic transversal velocity profiles. If, then, through an analysis of available observations, the existence of certain preferred atmospheric or oceanic current profiles is established, which profiles from a comparison with completely controlled laboratory experiments appear to be the result of frictional forces (lateral stresses), we are reasonably justified in assuming that the associated mass (solenoid) distribution in a transversal plane must be regarded as a result rather than as a cause of the motion. This point is stressed here since there seems to be a tendency on the part of many oceanographers to regard the mass distribution, which serves as a starting point in all dynamic calculations of so-called "convection" currents, as their cause. As a matter of fact, it is easy enough to show how, on a rotating globe, solenoids may be generated through mechanical means.[4] It is possible to develop criteria for the separation of such secondary *dynamic* solenoids from the *thermal* solenoids, which are the ultimate cause of all motion in the atmosphere. Thus one should expect to find the vertical correlation curve between temperature and salinity to be independent of location in an ocean current section whose solenoids are dynamic in origin. Similarly, in a section across a steady air current in which the solenoids are of secondary character, the vertical correlation between specific humidity and potential temperature ought to be reasonably constant. Illustrative examples will be furnished in the second part of this investigation.

B. Effect of the Earth's Rotation on Lateral Stresses

The evaluation of lateral stresses in the air or in the ocean brings up another problem of general significance, namely, the effect of curvature and of the earth's rotation on the turbulent exchange of momentum between fluid strata moving side by side. It thus forces us to choose between the "vorticity-transport" theory developed by Taylor[5] and the "momentum-transport" theory developed by Prandtl.[6] Taylor has pointed out that the structure of straight fluid current systems may be interpreted equally well with the aid of the one as with the aid of the other of these two theories but that, in the case of curved flow or flow in rotating systems the two theories lead to mutually exclusive results. It seems appropriate to follow up this comment of Taylor's with an analysis of the predictions of the two theories in as far as atmospheric and oceanic motion is concerned.

As a starting point we choose a steady terrestrial fluid system rotating cyclonically relative to the surface of the earth around a certain vertical axis A. The rotation of the earth itself may be resolved into a rotation around A and a rotation around an axis normal thereto. The latter rotation is without significance in the present connection. The relative linear velocity at a distance r from the axis is given by v. It we designate by

$f = 2\omega \sin L$ the Coriolis parameter, it follows that the absolute linear velocity V around the axis has the value

(2) $$V = v + \tfrac{1}{2} f r.$$

The absolute angular momentum around A is given by

(3) $$\Omega = rV = rv + \tfrac{1}{2} f r^2.$$

According to the momentum transfer theory each element displaced along the radius tends to retain its original angular momentum. Thus an element displaced from r to $r+l$ will produce, in its new position, a deviation of the observed angular momentum from the mean, given by

(4) $$\Omega' = \Omega_r - \Omega_{r+l} = -l \frac{\partial \Omega}{\partial r},$$

and consequently a deviation of the tangential velocity from the mean, given by

(5) $$v' = -\frac{l}{r} \frac{\partial \Omega}{\partial r}.$$

Assuming equipartition of eddy energy it follows that the shearing stress is given by

(6) $$\tau = -\rho\, \overline{u'v'} = \rho\, \overline{u'l} \frac{1}{r} \frac{\partial \Omega}{\partial r} = \rho \frac{l^2}{r^2} \left(\frac{\partial \Omega}{\partial r} \right)^2.$$

In this expression u' represents the radial (eddying) velocity. Thus the momentum transfer theory indicates that the shearing stress vanishes when the absolute angular momentum is independent of the distance from the axis. One may now introduce the relative motion in the above expression. The result is

(7) $$\tau = \rho l^2 \left(\frac{\partial V}{\partial r} + \frac{V}{r} \right)^2 = \rho l^2 \left(\frac{\partial v}{\partial r} + \frac{v}{r} + f \right)^2.$$

If the radius of curvature is sufficiently large, the above expression reduces to the form

(8) $$\tau = \rho l^2 \left(\frac{\partial v}{\partial x} + f \right)^2,$$

where x is a horizontal coordinate counted positive in a direction 90° to the right of the direction in which the current is flowing. Thus the momentum transfer theory indicates that in a straight air or ocean current the lateral shearing stresses vanish when the velocity decreases towards the right edge of the current at the rate

(9) $$\frac{\partial v}{\partial x} = -f.$$

This is a very steep rate, which in middle latitudes (43°) corresponds to a rate of shear of 1 cm.p.s. in 100 meters. Such horizontal rates of shear are hardly ever observed in the ocean and in the atmosphere they occur only along fronts. Thus, according to the momentum transfer theory, the right edge of a current always tends to accelerate the left