Modeling Planetary Turbulence at very high Reynolds number

A. Bracco* - J. von Hardenberg** - A. Provenzale*** - J. B. Weiss*,*

* Abdus Salam ICTP, Trieste
** CIMA, Università di Genova, Savona
*** ISAC-CNR, Torino
*,* PAOS, University of Colorado, Boulder, USA

DOI: 10.1388/SSC(2003)-AP-3

ABSTRACT – Vengono studiate le proprietà della populazione di vortici in flussi quasigeostrofici a numeri di Reynolds molto alti. Statistica, evoluzione e le proprietà di trasporto dei vortici coerenti vengono analizzate sia nel più semplice caso bidimensionale sia per un flusso quasigeostrofico completamente tridimensionale.

We investigate the properties of the vortex population in quasigeostrophic flows at very high Reynolds numbers. The statistics, the evolution and the transport properties of the coherent vortices are investigated in the simpler two-dimensional case and for a fully three-dimensional quasigeostrophic flow.

The dynamics of fluids on a rapidly rotating planet, such as Earth, is significantly influenced by the planetary rotation. When rotation is coupled with strong stable stratification, the vertical component of the velocity field is appreciably weaker than the two horizontal ones. As a consequence, the large scale motion of oceanic and atmospheric flows tends to become approximately two-dimensional on a local horizontal plane.

Oceanic and atmospheric flows are extremely turbulent and are characterized by a very high Reynolds number (the ratio between the advective and the viscous terms in the equations of motion of the fluid). Such flows are highly unpredictable in that a small uncertainty at a given initial time will amplify to render impossible a deterministic prediction of their future evolution.

An essential characteristic of rotating turbulent flows is the presence of coherent vortices, regions of space where the vorticity (the curl of the velocity field) is strongly concentrated and organized. Vortices are coherent when they keep their identity and shape for a time much longer than their typical local rotation time. The existence of these organized long-lived structures is in contrast with the common concept of turbulence as a random, disordered flow.

Coherent vortices are very common in nature. Geophysical examples include the stratospheric polar vortex; the large vortices on giant gaseous planets, such as the
Figure 1. – Panels (a-c) show the vorticity field at $t = 3$, $t = 7$ and $t = 11$ respectively. In this figure, the color map shades from red to blue represent positive and negative vorticity, respectively.

Great Red Spot on Jupiter; the polar dipole on Venus; the oceanic rings that develop in the Gulf Stream in the Atlantic Ocean and in the Kuroshio extention in the Pacific Ocean; the mid-depth salty eddies of Mediterranean water in the Atlantic Ocean; buoyant plumes in planetary boundary layers; and vortices generated by atmospheric flows past island orography, or by oceanic flows past undersea topography.

The simplest model which describes the evolution of stratified flows on a rapidly rotating sphere is the quasigeostrophic approximation (see Pedlosky 1987\(^1\) for a complete derivation of the equations and for details on the mathematics). This model assumes that the fluid is incompressible and that the viscosity is small.

Our project investigates the statistics, the evolution, and the transport properties of the coherent vortex population in quasigeostrophic flows at very high Reynolds numbers.

---

We present our work in two parts. First, we analyze the evolution of the vortex statistics in the simpler two-dimensional case, neglecting the vertical coordinate in the equation of motion. Two-dimensional turbulence is a zero-order approximation of the dynamics of large-scale rotating planetary flows. By ignoring the vertical component of the motion, we achieve a greater resolution on the horizontal plane, which permits the use of a lower viscosity coefficient and consequently of a higher Reynolds number.

The equation which describes two-dimensional turbulent flows has been integrated at the highest resolution ever reached in a direct integration. This simulation was done using a parallel, fully implicit multigrid elliptic solver. The given initial conditions are a homogeneous, isotropic, random field, which develops into a collection of coherent vortices surrounded by a turbulent background. In the background, the vorticity attains very low values, distributed around zero. In our case the flow is freely-decaying, i.e. there is no forcing term in the equation to balance the energy loss due to the diffusion nor to introduce new vorticity in the system. Once the vortex generation period is over, most of the vorticity from the

![Figure 2](image-url)  
**Figure 2.** Evolution of a QG flow starting from an initial isotropic random field. The red and blue surfaces represent positive and negative potential vorticity isosurfaces (PV) at PV = ±25.
initial field is concentrated inside the coherent structures and no new vortices form.

When two vortices of same sign are close enough they tend to rotate about each other and merge, so that the total number of coherent vortices decreases with time. Thus, the evolution proceeds through mutual vortex advection and strong inelastic interactions of same-sign vortices.

We focus on this intermediate, vortex-dominated stage, illustrated by Fig. 1, where the vorticity field is shown at different times of the evolution. In this stage, the analysis of the vortex population confirms the existence of temporal-scaling in the limit of infinite Reynolds number, as proposed by Carnevale et al. in 1991\(^2\). As predicted by the theory, the number of vortices as a function of time decreases as a power law. In addition, by comparing the one-point vorticity distribution at different times, we obtain strong evidence of the conservation of the vorticity amplitude inside the vortex cores, which constitutes the main hypothesis of the scaling theory. It is worth noting that past simulations did not provide any satisfactory confirmation of this hypothesis; our study revealed it due to the very high Reynolds number achieved\(^3\).

In the second part of this project we study the evolution of fully three-dimensional quasi-geostrophic flows initialized with isotropic, random initial conditions. The equation of motions are integrated on a high resolution grid (256\(^3\) grid points) in a periodic domain. The code employed is a parallel multigrid elliptic solver running on the Origin 2000 at CINECA. Figure 2 shows an example of the temporal evolution of the flow. As in the two-dimensional case, the vorticity gets organized into coherent

\[
\text{t} = 6.0 \quad \text{t} = 12.0 \quad \text{t} = 20.0
\]

\hspace{1cm}

\text{Figure 3. – Merging of two initial flat ellipsoidal potential vorticity distributions: potential vorticity on the central level at times t = 6, 12, 20.}


\(^3\) A. Bracco, J.C. McWilliams, G. Murante, A. Provenzale, and J.B. Weiss, \textit{Revisiting 2D turbulence at modern resolution}, Phys. of Fluids. (2000), 12 11, 2931 (The authors of these papers acknowledge support by CINECA).
structures and the evolution of the flow is dominated by vortex interactions such as horizontal merging and vertical alignment events\textsuperscript{4,5}. Figure 5 illustrates the merging of two initial flat ellipsoidal distributions of vorticity. In the three-dimensional case, the filaments of vorticity shed by the merging vortices may become unstable and roll-up to form secondary new vortices. This phenomenon has not been observed in two-dimensional simulations.

Coherent vortices have a strong influence on the transport properties of planetary turbulence, affecting the distribution of quantities such as energy, biological nutrients and pollutants. To better understand this phenomenon we study the transport properties of coherent vortices in quasi-geostrophic flows.

It is known that purely two-dimensional vortices are impermeable to inward and outward fluxes of passive tracers. Particles or other passive materials can be trapped inside vortex cores for long times and can be transported by the vortex over long distances (see Provenzale, 1999 for a review on this subject)\textsuperscript{6}.

To investigate the transport properties of three-dimensional, quasi-geostrophic vortices, we distribute a large number \((N = 49152)\) of neutral Lagrangian particles (i.e. particles having the same density of the fluid) on three different levels

\textsuperscript{4} J.C. McWilliams and J.B. Weiss, \textit{Anisotropic geophysical vortices}, Chaos (1994), 4, 305.

\textsuperscript{5} J. von Hardenberg, J.C. McWilliams, A. Provenzale, A. Shchepetkin and J.B. Weiss, \textit{Vortex Merging in Quasigeostrophic Flows}, J. Fluid Mech (2000), 412, 331 (The authors of these papers acknowledge support by CINECA).

of the flow. Neutral particles are advected by the flow and their velocities are equal to the velocity of the flow at the particle locations. We study how the particles disperse as a function of their initial location and the trapping of the particles by the vortices. Figure 6 shows how such groups of particles seeded at time $t = 5$ have evolved at time $t = 20$. The results indicate that the dispersion properties of the flow do not differ significantly from the two-dimensional case [1]. However, differences in the ability by vortex cores to trap particles at different heights with respect the vortex center have been observed.

Massively parallel computers have allowed us to investigate planetary turbulence at very high Reynolds numbers. We have, for the first time, been able to observe the detailed behavior of the coherent vortices present in these flows. A deeper understanding of the dynamics of coherent vortices will ultimately lead to a better understanding of the climate of the Earth.

**Publications**